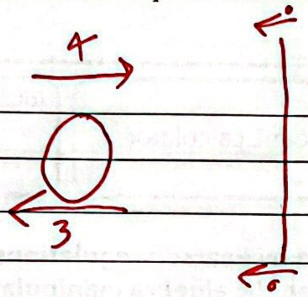
1. Two objects, A of mass 18 kg and B of mass 7 kg, are moving in the same straight line on a smooth horizontal surface. Initially, they are moving with the same speed of 4 ms^{-1} and in the same direction. Object B collides with a vertical wall which is perpendicular to its direction of motion and rebounds with a speed of 3 ms^{-1}. Subsequently, the two objects A and B collide directly. The coefficient of restitution between the two objects is $\frac{5}{7}$.

(a) Find the coefficient of restitution between B and the wall. **(1)**

(b) Determine the speed of A and the speed of B immediately after the two objects collide. **(7)**

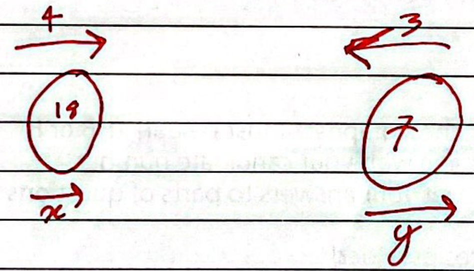
(c) Calculate the impulse exerted by A on B due to the collision and clearly state its units. **(2)**

a)



$e = \frac{3-0}{4-0} = \frac{3}{4}$

b)



NLB

$\frac{y-x}{7} = \frac{5}{7}$

$\therefore y - x = 5$

CLM

$4 \times 18 + 7(-3) = 18x + 7y$

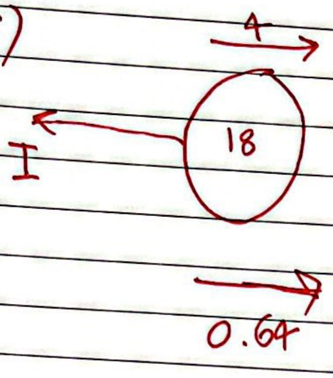
$18x + 7y = 51$

$\therefore x = \frac{16}{25} = 0.64 \, m/s$

$y = \frac{141}{25} = 5.64 \, m/s$

Question 1 continued

c)



$I = 18(-0.64 - -4)$

$= 60.48\ Ns$

2.

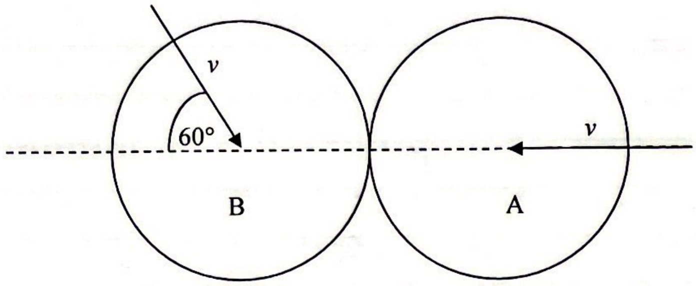
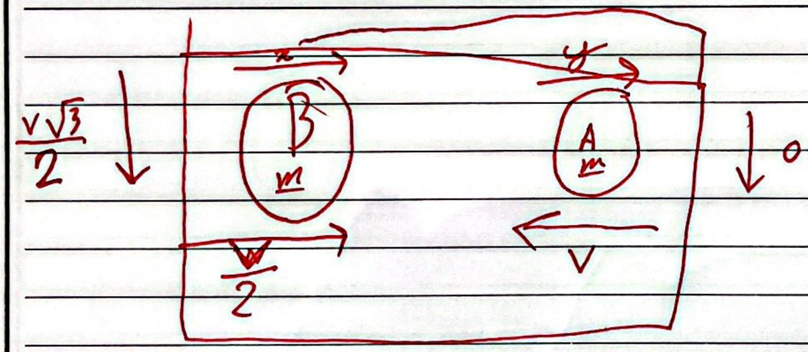


Figure 1

Figure 1 shows the instant of impact of two identical uniform smooth spheres, A and B, each with mass m. Immediately before they collide, the spheres are sliding towards each other on a smooth horizontal table in the directions shown in the diagram, each with speed v. The coefficient of restitution between the spheres is $\frac{1}{2}$.

Show that, immediately after the collision, the speed of A is $\frac{1}{8}v$ and find its direction of motion.

(6)



NLR

$$\frac{y-x}{\frac{3v}{2}} = \frac{1}{2}$$

$$y - x = \frac{3v}{4}$$

$$\therefore x = \frac{-5v}{8}$$

CLM

$$\frac{mv}{2} - mv = mx + my$$

$$-\frac{v}{2} = x + y$$

$$y = \frac{v}{8}$$

East Direction

Speed of A $= \sqrt{\left(\frac{v}{8}\right)^2 + 0^2}$

$= v/8$

3. A car of mass 1200 kg has an engine that is capable of producing a maximum power of 80 kW. When in motion, the car experiences a constant resistive force of 2000 N.

a) Calculate the maximum possible speed of the car when travelling on a straight horizontal road.

(3)

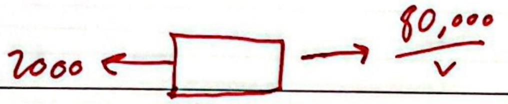
b) The car travels up a slope inclined at an angle α to the horizontal, where $\sin \alpha = \frac{1}{20}$

If the car's engine is working at 80% capacity, calculate the acceleration of the car at the instant when its speed is 20m/s.

(5)

c) Explain why the assumption of a constant resistive force may be unrealistic.

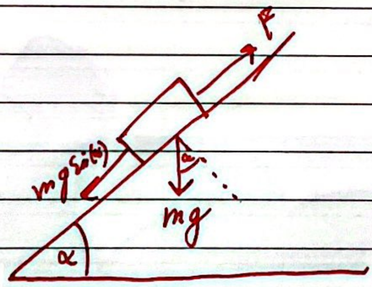
(1)



$(a=0)$

$\therefore \frac{80{,}000}{v} = 2000$ So $\frac{80000}{2000} = v = 40\,m/s$

b)



$\sin(\alpha) = \frac{1}{20}$

$\cos(\alpha) = \frac{\sqrt{399}}{20}$

$V = 20$

$P = 64\,kW$

So $F = \frac{64000}{20} = 3200$

$3200 - mg\sin(\alpha) - 2000 = ma$

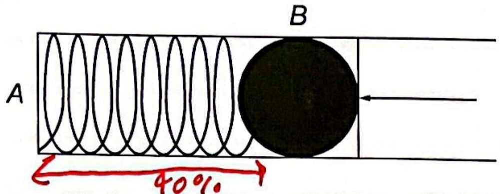
$1200 - \frac{1200g}{20} = 1200a$

$1200 - 60g = 1200a$

$\frac{51}{100} = a = 0.51\,m/s^2$

c) It changes depending on surface & speed.

4. The diagram shows a spring of natural length 0.15 m enclosed in a smooth horizontal tube. One end of the spring A is fixed and the other end B is compressed against a ball of mass 0.1 kg.



Initially, the ball is held in equilibrium by a force of 21 N so that the compressed length of the spring is 40% of its natural length.

(a) Calculate the modulus of elasticity of the spring.

(3)

(b) The ball is released by removing the force. Determine the speed of the ball when the spring reaches its natural length. Give your answer correct to two significant figures.

(5)

a) $F = \frac{\lambda x}{l}$ $\quad \therefore \; 21 = \frac{\lambda(0.09)}{0.15}$ → 60% of 0.15m

$\therefore \lambda = 35$

b) $\frac{35(0.09)^2}{2 \times 0.15} = \frac{1}{2}(0.1)v^2$

$9.45 = \frac{v^2}{2}$

$18.9 = v^2$ $\quad$ so $v = 4.347\ldots$

$= 4.3$ m/s (2.s.f)

5. Three spheres A, B, C, of equal radii and each of mass m kg, lie at rest on a smooth horizontal surface such that their centres are in a straight line with B between A and C. The coefficient of restitution between A and B is e. Sphere A is projected towards B with speed u ms^{-1} so that it collides with B.

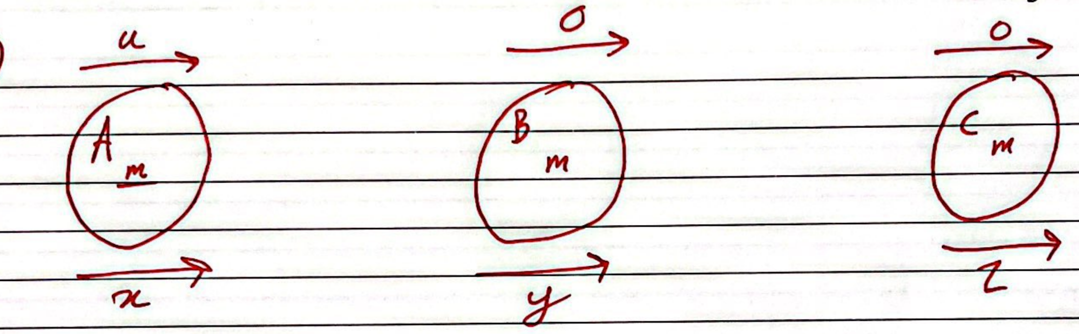
(a) Find expressions, in terms of e and u, for the speed of A and the speed of B after they collide. **(7)**

You are now given that $e = \frac{1}{2}$

(b) Find, in terms of m and u, the loss in kinetic energy due to the collision between A and B. **(2)**

(c) After the collision between A and B, sphere B then collides with C. The coefficient of restitution between B and C is e_1. Show that there will be no further collisions if $e_1 \leqslant \frac{1}{3}$ **(3)**

a)



NLR

$$\frac{y-x}{u} = e$$

$\therefore y - x = eu$ (1)

CLM

$mu = mx + my$

$\therefore u = x + y$ (2)

(1) → (2)

$u - x - x = eu$

$\therefore x = \frac{u - eu}{2}$

$y = u - x$

$= \frac{2u}{2} - \frac{u - eu}{2}$

$= \frac{u + eu}{2}$

Question 5 continued

b) If $e = \frac{1}{2}$

$$x = \frac{u(1-e)}{2} = \frac{u}{4}$$

$$y = \frac{u(1+e)}{2} = \frac{3u}{4}$$

Hence, energy before is:

$$\frac{1}{2} m u^2$$

Energy After:
$$\frac{1}{2} m \left(\frac{u}{4}\right)^2 + \frac{1}{2} m \left(\frac{3u}{4}\right)^2$$

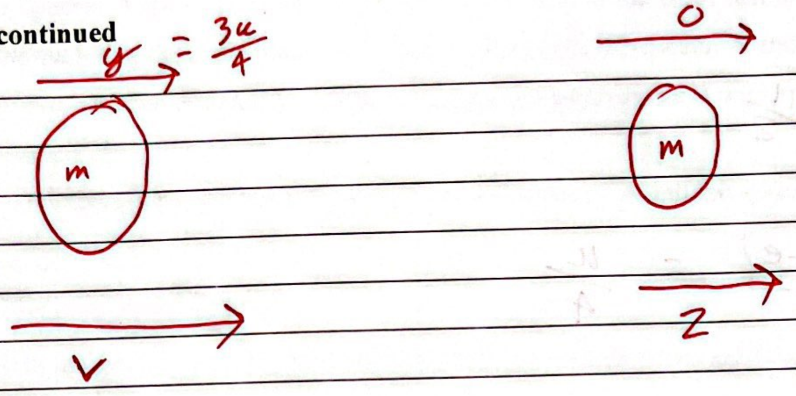
$$= \frac{1}{2} m \left(\frac{u^2}{16}\right) + \frac{1}{2} m \left(\frac{9u^2}{16}\right)$$

$$= \frac{1}{2} m \left(\frac{10u^2}{16}\right) = \frac{5mu^2}{16}$$

Energy Loss: $\frac{1}{2} mu^2 - \frac{5mu^2}{16} = \frac{3mu^2}{16}$

Question 5 continued

c)



$v \geq x$ For no further collision.

C is already infront B so $z \geq v$. No need to verify

Now, if $v \geq x$ we need v

NLR	CLM
$e_1 = \dfrac{z - v}{\frac{3u}{4}}$	$\dfrac{3um}{4} = mv + mz$
$\dfrac{3e_1 u}{4} = z - v$	$\dfrac{3u}{4} = v + z$

$v = \dfrac{3u - 3e_1 u}{8}$ & $z = \dfrac{3e_1 u + 3u}{8}$

If $\dfrac{3u - 3e_1 u}{8} \geq \dfrac{u}{4}$ $\therefore 3u - 3e_1 u \geq 2u$ $\therefore u \geq 3e_1 u$

So $\frac{1}{3} \geq e_1$

6. The fixed points A and B lie on a line of greatest slope of a smooth inclined plane, with B higher than A. The horizontal distance from A to B is 2.4 m and the vertical distance is 0.7 m. The fixed point C is 2.5 m vertically above B. A light elastic string of natural length 2.2 m has one end attached to C and the other end attached to a small block of mass 9 kg which is in contact with the plane. The block is in equilibrium when it is at A, as shown in Figure 2.

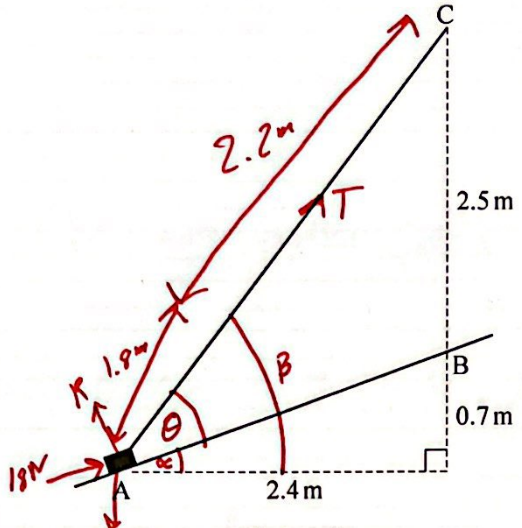


Figure 2

9g

(a) Show that the modulus of elasticity of the string is 37.73 N.

(5)

The block starts at A and is at rest. A constant force of 18 N, acting in the direction AB, is then applied to the block so that it slides along the line AB.

(b) Find the magnitude and direction of the acceleration of the block,

(i) when it leaves the point A,

(ii) when it reaches the point B.

(6)

(c) Find the speed of the block when it reaches the point B.

(6)

$\theta = \beta - \alpha$ So $\cos(\theta) = \cos(\beta - \alpha) = \cos(\beta)\cos(\alpha) + \sin(\beta)\sin(\alpha)$

$= \frac{2.4}{4} \times \frac{2.4}{2.5} + \frac{3.2}{4} \times \frac{0.7}{2.5}$

$= 0.8$

Question 6 continued

B

$0.8T = 9g\,Si(\alpha)$

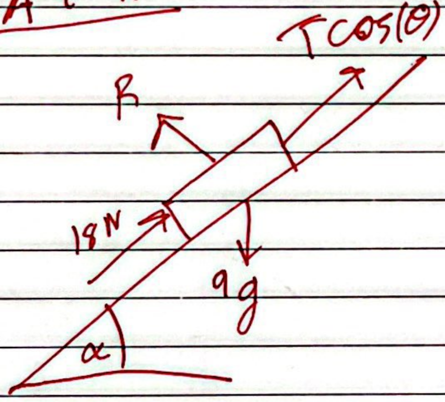
$0.8T = 9g \times \frac{0.7}{2.5}$

$\therefore T = 30.87$

$30.87 = \frac{1.8\lambda}{2.2}$

$\therefore \lambda = 37.73N$

b) <u>At A</u>

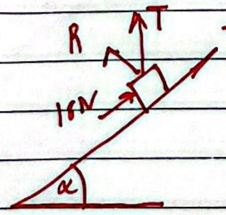


$0.8T + 18 - 9g\,Sin(\alpha) = ma$

$\therefore (30.87)\times 0.8 + 18 - 9g(0.28) = 9a$

$\therefore a = 2\,m/s^2$ up the slope.

At B



$\rightarrow T = \frac{37.73(0.3)}{2.2} = 5.145$

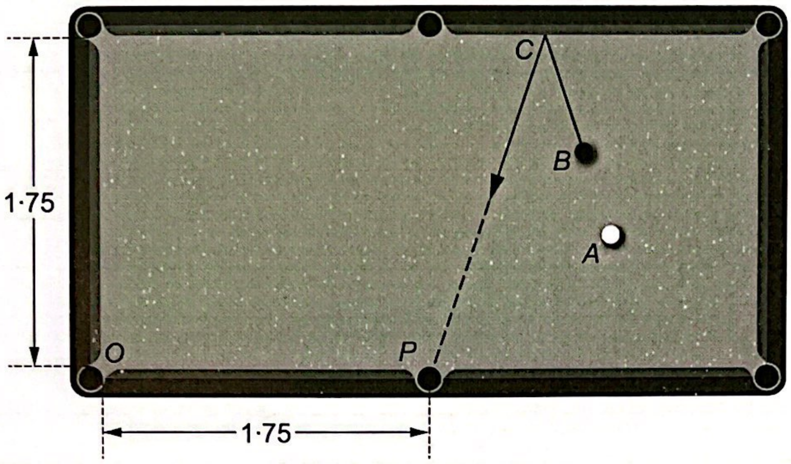
$\therefore 5.145 \times Sin(\alpha) - 9g\,Sin(\alpha) + 18 = 9a$

$\therefore a = -0.58\,m/s^2$

0.58 m/s² down the plane

7. Ryan is playing a game of snooker. The horizontal table is modelled as the horizontal x-y plane with the point O as the origin and unit vectors parallel to the x-axis and the y-axis denoted by **i** and **j** respectively. All balls on the table have a common mass m kg. The table and the four sides, called cushions, are modelled as smooth surfaces.

The dimensions of the table, in metres, are as shown in the diagram.



Initially, all balls are stationary. Ryan strikes ball A so that it collides with ball B. Before the collision, A has velocity $(-\mathbf{i} + 8\mathbf{j})$ ms^{-1} and, after the collision, it has velocity $(2\mathbf{i} + \mathbf{j})$ ms^{-1}.

(a) Show that the velocity of ball B after the collision is $(-3\mathbf{i} + 7\mathbf{j})$ ms^{-1}. **(3)**

After the collision with ball A, ball B hits the cushion at point C before rebounding and moving towards the pocket at P. The cushion is parallel to the vector **i** and the coefficient of restitution is between the cushion and ball B is $\frac{5}{7}$.

(b) Calculate the velocity of ball B after impact with the cushion. **(3)**

(c) Find, in terms of m, the magnitude of the impulse exerted on ball B by the cushion at C, stating your units clearly. **(3)**

(d) Given that C has position vector $(x\mathbf{i} + 1.75\mathbf{j})$ m,

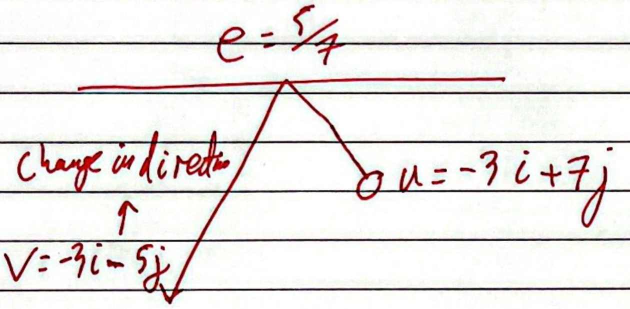
(i) determine the time taken between the ball hitting the cushion at C and entering the pocket at P,
(ii) find the value of x. **(4)**

a) $m(-i + 8j) = m(2i + j) + m(v)$

$v = (-3i + 7j)$ m/s

Question 7 continued

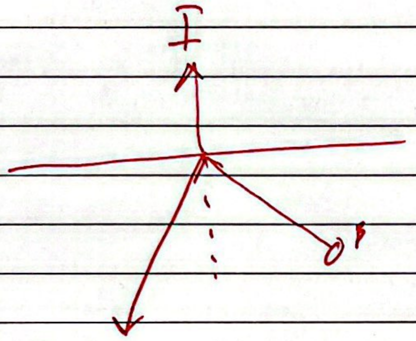
b)



NLR Perpendicular to plane

$\frac{5}{7} = \frac{u}{7} \quad \therefore u = -5$

C)



Impulse perpendicular to plane

$I = m(-5 - 7)j$

$I = -12mj$

$\therefore |I| = 12m$ Ns

d)

$r = (xi + 1.75j) + t(-3i - 5j)$

$(1.75i + 0j) = (x - 3t)i + (1.75 - 5t)j$

$1.75 - 5t = 0 \quad \therefore t = \frac{1.75}{5} = 0.35s$

$x = 1.75 + 3(0.35) = 2.8$