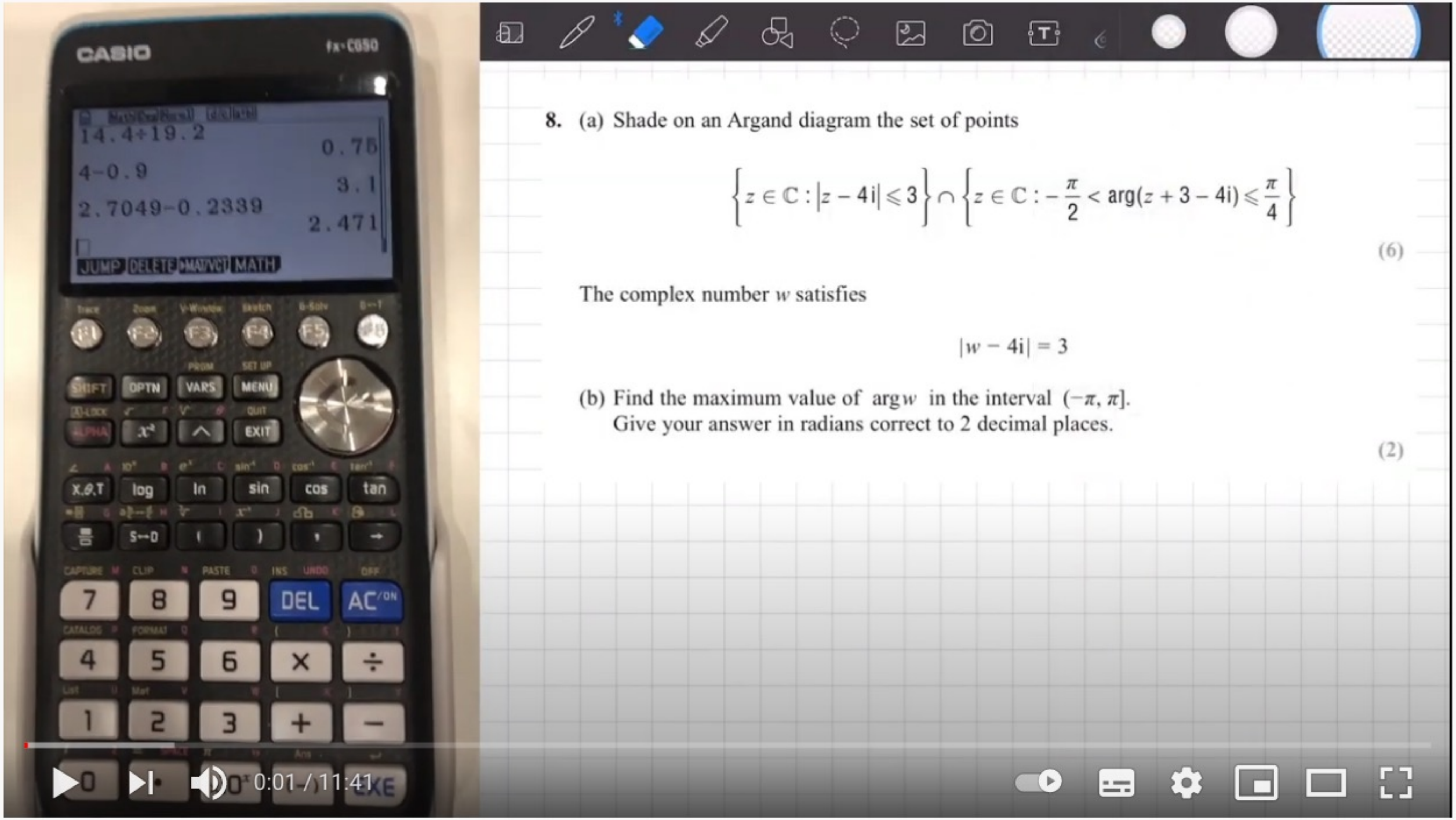
Mr. Chan’s Loci in Argand Diagram
Questions by topic pack

https://www.youtube.com/watch?v=XBrVET4gl



12Fm Core Pure - Loci in Argand Diagrams

Difficulty #1 (Circles, Half Lines, Perpendicular Bisector), Equate, Regions (Old Spec OCR FP1)

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Difficulty #2 (Circles, Half Lines, Perpendicular Bisector), Equate, Regions (Old Spec AQA FP2) Include (max/min)

- 38. AQA Jan 2006 FP2
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OCR JUNE 2005

6 The loci C_1 and C_2 are given by

$$|z - 2\mathrm{i}| = 2 \qquad \text{and} \qquad |z + 1| = |z + \mathrm{i}|$$

respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. [5]

(ii) Hence write down the complex numbers represented by the points of intersection of C_1 and C_2. [2]

6.	(i) Circle Centre (0, 2) Radius 2 Straight line Through origin with positive slope	B1 B1 B1 B1 B1	5	Sketch(s) showing correct features, each mark independent
	(ii) 0 or 0 +0i and 2 + 2i	B1ftB1ft	2	Obtain intersections as complex numbers
			7	

OCR JAN 2006

7 **(a)** The complex number $3 + 2i$ is denoted by w and the complex conjugate of w is denoted by w^*. Find

(i) the modulus of w, [1]

(ii) the argument of w^*, giving your answer in radians, correct to 2 decimal places. [3]

(b) Find the complex number u given that $u + 2u^* = 3 + 2i$. [4]

(c) Sketch, on an Argand diagram, the locus given by $|z + 1| = |z|$. [2]

7.	(a) (i) $\sqrt{13}$	B1	1	Obtain correct answer, decimals OK
	(ii)			
		M1		Using $\tan^{-1} {}^{b}/_{a}$, or equivalent trig allow + or -
	- 0.59	A1		Obtain 0.59
		A1	3	Obtain correct answer
	(b)			
		M1		Express LHS in Cartesian form & equate real and imaginary parts
	1 – 2i	A1A1		Obtain $x = 1$ and $y = -2$
		A1	4	Correct answer written as a complex number
	(c)	B1		Sketch of vertical straight line
		B1	2	Through (- 0.5, 0)
			10	

OCR JUNE 2006

6 In an Argand diagram the loci C_1 and C_2 are given by

$$|z| = 2 \qquad \text{and} \qquad \arg z = \tfrac{1}{3}\pi$$

respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. [5]

(ii) Hence find, in the form $x + \mathrm{i}y$, the complex number representing the point of intersection of C_1 and C_2. [2]

6..	(i) Circle , Centre *O* radius 2 One straight line Through *O* with +ve slope In 1st quadrant only	B1 B1 B1 B1 B1	 5	Sketch showing correct features
	(ii) $1 + i\sqrt{3}$	M1 A1	 2 **7**	Attempt to find intersections by trig, solving equations or from graph Correct answer stated as complex number

OCR JAN 2007

4 **(i)** Sketch, on an Argand diagram, the locus given by $|z - 1 + \mathrm{i}| = \sqrt{2}$. [3]

(ii) Shade on your diagram the region given by $1 \leqslant |z - 1 + \mathrm{i}| \leqslant \sqrt{2}$. [3]

4.	(i)	B1		Circle
		B1		Centre (1, -1)
		B1	3	Passing through (0, 0)
	(ii)	B1		Sketch a concentric circle
		B1		Inside (i) and touching axes
		B1	3	Shade between the circles

OCR JUNE 2007

8 The loci C_1 and C_2 are given by $|z-3|=3$ and $\arg(z-1)=\frac{1}{4}\pi$ respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. [6]

(ii) Indicate, by shading, the region of the Argand diagram for which

$$|z-3| \leqslant 3 \quad \text{and} \quad 0 \leqslant \arg(z-1) \leqslant \tfrac{1}{4}\pi.$$ [2]

8	(i) Circle, centre (3, 0), y-axis a tangent at origin Straight line, through (1, 0) with +ve slope In 1st quadrant only (ii) Inside circle, below line, above x-axis	B1B1 B1 B1 B1 B1 B2ft	 6 2 **8**	Sketch showing correct features N.B. treat 2 diagrams asa MR Sketch showing correct region SR: B1ft for any 2 correct features

OCR JAN 2008

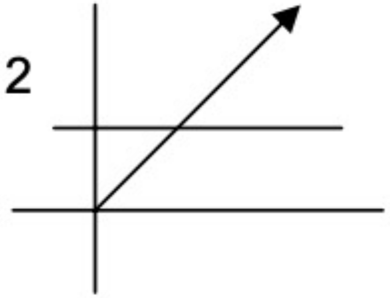
6 The loci C_1 and C_2 are given by

$$|z| = |z - 4\text{i}| \qquad \text{and} \qquad \arg z = \tfrac{1}{6}\pi$$

respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. [5]

(ii) Hence find, in the form $x + \text{i}y$, the complex number represented by the point of intersection of C_1 and C_2. [3]

6	(i) 	B1 B1 B1 B1 B1	 5	Horizontal straight line in 2 quadrants Through (0, 2) Straight line Through O with positive slope In 1st quadrant only
	(ii) $2\sqrt{3} + 2\mathrm{i}$	B1 M1 A1	 3 **8**	State or obtain algebraically that $y = 2$ Use suitable trigonometry Obtain correct answer a.e.f. decimals OK must be a complex number

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OCR June 2008

2 The complex number $3 + 4i$ is denoted by a.

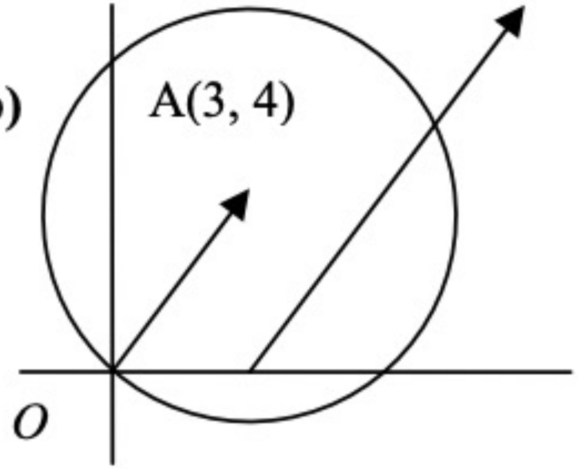
(i) Find $|a|$ and $\arg a$. [2]

(ii) Sketch on a single Argand diagram the loci given by

(a) $|z - a| = |a|$, [2]

(b) $\arg(z - 3) = \arg a$. [3]

2	(i)	5	B1	Correct modulus
		0.927 or 53.1°	B1	Correct argument, any equivalent form
			2	
	(ii)(a)		B1	Circle centre A (3, 4)
			B1	Through O, allow if centre is (4, 3)
			2	
	(b)		B1	Half line with +ve slope
			B1	Starting at (3, 0)
			B1	Parallel to OA, (implied by correct arg shown)
			3	



OCR JAN 2009

10 **(i)** Use an algebraic method to find the square roots of the complex number $2 + i\sqrt{5}$. Give your answers in the form $x + iy$, where x and y are exact real numbers. **[6]**

(ii) Hence find, in the form $x + iy$ where x and y are exact real numbers, the roots of the equation

$$z^4 - 4z^2 + 9 = 0.$$ **[4]**

(iii) Show, on an Argand diagram, the roots of the equation in part **(ii)**. **[1]**

(iv) Given that α is the root of the equation in part **(ii)** such that $0 < \arg \alpha < \frac{1}{2}\pi$, sketch on the same Argand diagram the locus given by $|z - \alpha| = |z|$. **[3]**

10	**(i)** $x^2 - y^2 = 2, 2xy = \sqrt{5}$	M1 A1		Attempt to equate real and imaginary parts Obtain both results a.e.f.
	$4x^4 - 8x^2 - 5 = 0$	M1 M1		Eliminate to obtain quadratic in x^2 or y^2 Solve to obtain x (or y) values
	$x = \pm\frac{\sqrt{10}}{2}, y = \pm\frac{\sqrt{2}}{2}$	A1		Correct values for both x & y obtained a.e.f.
	$\pm(\frac{\sqrt{10}}{2} + \mathrm{i}\frac{\sqrt{2}}{2})$	A1	6	Correct answers as complex numbers
	(ii) $z^2 = 2 \pm \mathrm{i}\sqrt{5}$	M1 A1		Solve quadratic in z^2 Obtain correct answers
	$z = \pm(\frac{\sqrt{10}}{2} \pm \mathrm{i}\frac{\sqrt{2}}{2})$	M1 A1ft	4	Use results of (i) Obtain correct answers, ft must include root from conjugate
	(iii)	B1ft	1	Sketch showing roots correctly
	(iv)	B1 B1ft B1ft	3 **14**	Sketch of straight line, $\perp$ to α Bisector

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OCR JUNE 2009

6 The complex number $3 - 3\mathrm{i}$ is denoted by a.

(i) Find $|a|$ and $\arg a$. **[2]**

(ii) Sketch on a single Argand diagram the loci given by

(a) $|z - a| = 3\sqrt{2}$, **[3]**

(b) $\arg(z - a) = \frac{1}{4}\pi$. **[3]**

(iii) Indicate, by shading, the region of the Argand diagram for which

$$|z - a| \geqslant 3\sqrt{2} \quad \text{and} \quad 0 \leqslant \arg(z - a) \leqslant \tfrac{1}{4}\pi.$$ **[3]**

6.	(i) $3\sqrt{2}, -\frac{\pi}{4}$ or -45° AEF	B1 B1	2	State correct answers
	(ii)(a)	B1B1 B1 ft	3	Circle, centre (3, -3), through O ft for $(\pm 3, \pm 3)$ only
	(ii)(b)	B1 B1 B1	3	Straight line with +ve slope, through (3, -3) or their centre Half line only starting at centre
	(iii)	B1ft B1ft B1ft	3 **11**	Area above horizontal through a, below (ii) (b) Outside circle

OCR JAN 2010

8 The complex number a is such that $a^2 = 5 - 12\text{i}$.

(i) Use an algebraic method to find the two possible values of a. **[5]**

(ii) Sketch on a single Argand diagram the two possible loci given by $|z - a| = |a|$. **[4]**

8	**(i)**	M1		Attempt to equate real and imaginary parts of $(x + iy)^2$ & $5 - 12i$
	$x^2 - y^2 = 5$ and $xy = -6$	A1		Obtain both results, a.e.f
		M1		Obtain quadratic in x^2 or y^2
		M1		Solve to obtain $x = (\pm)3$ or $y = (\pm)2$
	$\pm(3 - 2i)$	A1	**5**	Obtain correct answers as complex nos
	(ii)	B1ft		Circle with centre at their square root
		B1		Circle passing through origin
		B1ft		2^{nd} circle centre correct relative to 1^{st}
		B1	**4**	Circle passing through origin
		9		

OCR JUNE 2010

6 **(i)** Sketch on a single Argand diagram the loci given by

(a) $|z - 3 + 4i| = 5$, **[2]**

(b) $|z| = |z - 6|$. **[2]**

(ii) Indicate, by shading, the region of the Argand diagram for which

$$|z - 3 + 4i| \leqslant 5 \quad \text{and} \quad |z| \geqslant |z - 6|.$$ **[2]**

6	(i)	(a)	B1B1	**2**	Circle centre (3, –4), through origin
		(b)	B1B1	**2**	Vertical line, clearly $x = 3$
	(ii)		B1ft		Inside their circle
			B1ft	**2**	And to right of their line, if vertical
				6	

OCR JAN 2011

6 **(i)** Sketch on a single Argand diagram the loci given by

(a) $|z| = |z - 8|$, **[2]**

(b) $\arg(z + 2\text{i}) = \frac{1}{4}\pi$. **[3]**

(ii) Indicate by shading the region of the Argand diagram for which

$$|z| \leqslant |z - 8| \quad \text{and} \quad 0 \leqslant \arg(z + 2\text{i}) \leqslant \frac{1}{4}\pi.$$ **[3]**

6	(i)	(a)	B1*	Vertical line
			depB1 **2**	Clearly through (4, 0)
		(b)	B1	Sloping line with +ve slope
			B1	Through (0, -2)
			B1ft **3**	Half line starting on *y*-axis 45° shown convincingly
	(ii)		B1ft	Shaded to left of their **(i) (a)**
			B1ft	Shaded below their **(i) (b)** must be +ve slope
			B1ft **3**	Shaded above horizontal through their (0, -2)
				NB These 3 marks are independent, but 3/3 only for fully correct answer.
			8	

OCR JUNE 2011

5 The complex number $1 + i\sqrt{3}$ is denoted by a.

(i) Find $|a|$ and $\arg a$. **[2]**

(ii) Sketch on a single Argand diagram the loci given by $|z - a| = |a|$ and $\arg(z - a) = \frac{1}{2}\pi$. **[6]**

5	(i)	$\lvert a \rvert = 2$	B1		Correct modulus
		$\arg a = 60°, \frac{\pi}{3}, 1.05$	B1	**2**	Correct argument
	(ii)		B1		Circle
			B1		Centre $(1, \sqrt{3})$
			B1		Through origin, centre $(\pm 1, \pm\sqrt{3})$ and another y intercept
			B1		Vertical line
			B1*		Through a or their centre, with +ve gradient
			DB1		Correct half line
				6	
			8		

OCR JAN 2012

6 Sketch, on a single Argand diagram, the loci given by $|z - \sqrt{3} - \mathrm{i}| = 2$ and $\arg z = \frac{1}{6}\pi$. **[6]**

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6				B1 B1 B1 B1 B1 B1 **[6]**	Circle Centre $(\sqrt{3},1)$ Passing through O and crosses y-axis again Line, with correct slope shown $\frac{1}{2}$ line starting at O Completely correct diagram for both loci	Ignore shading

OCR JUNE 2012

7 The loci C_1 and C_2 are given by $|z - 3 - 4i| = 4$ and $|z| = |z - 8i|$ respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. **[6]**

(ii) Hence find the complex numbers represented by the points of intersection of C_1 and C_2. **[2]**

(iii) Indicate, by shading, the region of the Argand diagram for which

$$|z - 3 - 4i| \leqslant 4 \text{ and } |z| \geqslant |z - 8i|.$$ **[2]**

Question			Answer	Marks	Guidance	
7	(i)			B1B1 B1ft B1ft B1B1 **[6]**	Circle, centre (3 , 4) Touching *x*-axis, ft for (3, −4) etc as centre Crossing *y*-axis twice Horizontal line, *y* intercept 4	
7	(ii)		–1 +4i 7 + 4i	B1B1 **[2]**	State correct answers	
7	(iii)			B1ft B1 **[2]**	Inside circle or above line Completely correct diagram	

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OCR JAN 2013

7 **(i)** Sketch on a single Argand diagram the loci given by

(a) $|z| = 2$, **[2]**

(b) $\arg(z - 3 - \text{i}) = \pi$. **[3]**

(ii) Indicate, by shading, the region of the Argand diagram for which

$$|z| \leqslant 2 \quad \text{and} \quad 0 \leqslant \arg(z - 3 - \text{i}) \leqslant \pi.$$ **[2]**

Question			Answer	Marks	Guidance
7	(i)	(a)		B1 B1 **[2]**	Circle Centre *O* and radius 2
7	(i)	(b)		B1 B1 B1 **[3]**	Horizontal line (3, 1) on their line ½ line to left i.e. horizontal
7	(ii)			B1 B1 **[2]**	Shade only inside their circle or above their horizontal line Completely correct diagram

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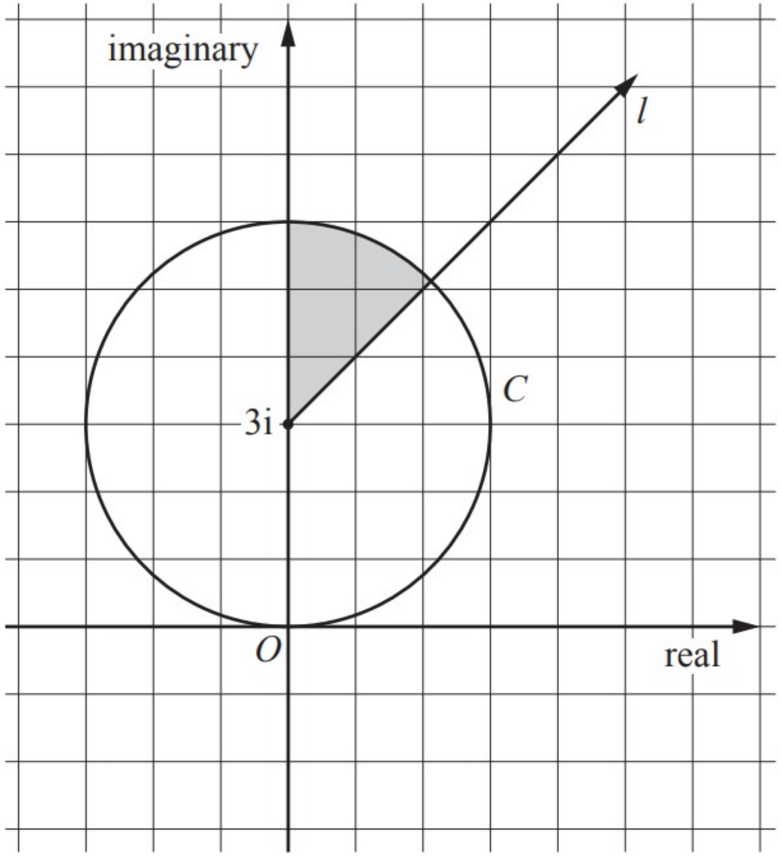
OCR JUNE 2013

6



The Argand diagram above shows a half-line l and a circle C. The circle has centre 3i and passes through the origin.

(i) Write down, in complex number form, the equations of l and C. **[4]**

(ii) Write down inequalities that define the region shaded in the diagram. [The shaded region includes the boundaries.] **[3]**

6	**(i)**			M1	Use arg $(z - \text{a}) = \theta$ in equation for l condone missing brackets
			$\arg(z-3i)=\frac{1}{4}\pi$	A1	Obtain correct answer
				M1	Use $\lvert z-a \rvert = k$ in equation for C, k must be real
			$\lvert z-3i \rvert = 3$	A1	Obtain correct answer
				[4]	
	(ii)		$\lvert z-3i \rvert \le 3$ or e.g. $x^2+(y-3)^2 \le 9$	B1	Obtain correct inequality, or answer consistent with sensible **(i)**
			$\frac{1}{4}\pi \le \arg(z-3i) \le \frac{1}{2}\pi$ or $y \ge x+3$, $x \ge 0$	B1 B1	Each correct single inequality, or answer consistent with sensible **(i)**
				[3]	**SC if < used consistently, but otherwise all correct, B2**

AQA Jan 2006 FP2

3 The complex numbers z_1 and z_2 are given by

$$z_1 = \frac{1+\mathrm{i}}{1-\mathrm{i}} \qquad \text{and} \qquad z_2 = \frac{1}{2} + \frac{\sqrt{3}}{2}\mathrm{i}$$

(a) Show that $z_1 = \mathrm{i}$. *(2 marks)*

(b) Show that $|z_1| = |z_2|$. *(2 marks)*

(c) Express both z_1 and z_2 in the form $r\mathrm{e}^{\mathrm{i}\theta}$, where $r > 0$ and $-\pi < \theta \leqslant \pi$. *(3 marks)*

(d) Draw an Argand diagram to show the points representing z_1, z_2 and $z_1 + z_2$. *(2 marks)*

(e) Use your Argand diagram to show that

$$\tan \frac{5}{12}\pi = 2 + \sqrt{3}$$

(3 marks)

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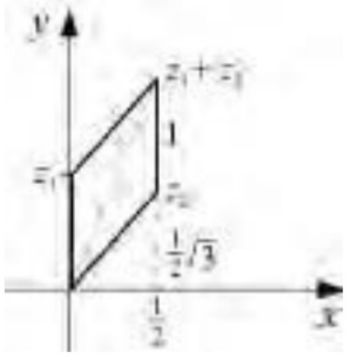
MFP2 (cont)

Q	Solution	Marks	Total	Comments
3(a)	$\frac{1+i}{1-i}=\frac{(1+i)^2}{1-i^2}=i$	M1A1	2	AG
(b)	$\lvert z_2\rvert=\sqrt{\frac{1}{4}+\frac{3}{4}}=1=\lvert z_1\rvert$	M1A1	2	
(c)	$r=1$	B1		PI
	$\theta=\frac{1}{2}\pi,\ \frac{1}{3}\pi$	B1B1	3	Deduct 1 mark if extra solutions
(d)		B2,1F	2	Positions of the 3 points relative to each other, must be approximately correct
(e)	$\text{Arg}(z_1+z_2)=\frac{5}{12}\pi$	B1		**Clearly** shown
	$\tan\frac{5}{12}\pi=\dfrac{1+\frac{1}{2}\sqrt{3}}{\frac{1}{2}}$	M1		Allow if BO earned
	$=2+\sqrt{3}$	A1	3	AG must earn BO for this
	Total		**12**	

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AQA Jan 2006 FP2

5 The complex number z satisfies the relation

$$|z + 4 - 4\text{i}| = 4$$

(a) Sketch, on an Argand diagram, the locus of z. *(3 marks)*

(b) Show that the greatest value of $|z|$ is $4(\sqrt{2} + 1)$. *(3 marks)*

(c) Find the value of z for which

$$\arg(z + 4 - 4\text{i}) = \tfrac{1}{6}\pi$$

Give your answer in the form $a + \text{i}b$. *(3 marks)*

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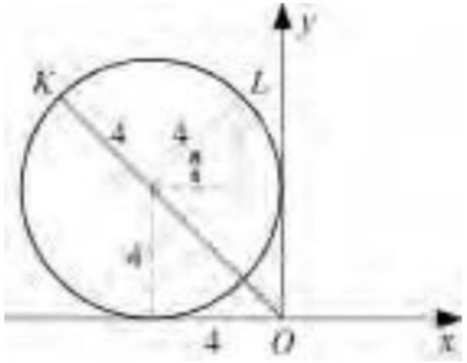
MFP2 (cont)

Q	Solution	Marks	Total	Comments
5(a)		B1 B1 B1	3	Circle Correct centre Touching both axes
(b)	$\|z\|\max = OK$	M1		Accept $\sqrt{4^2+4^2}+4$ as a method
	$=\sqrt{4^2+4^2}+4$	A1F		Follow through circle in incorrect position
	$=4\left(\sqrt{2}+1\right)$	A1F	3	AG
(c)	Correct position of z, ie L	M1		
	$a=-\left(4-4\cos\frac{1}{6}\pi\right)$			
	$=-\left(4-2\sqrt{3}\right)$	A1F		Follow through circle in incorrect position
	$b=4+4\sin\frac{1}{6}\pi=6$	A1F	3	
	Total		9	

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AQA June 2006 FP2

4 (a) On one Argand diagram, sketch the locus of points satisfying:

(i) $|z - 3 + 2i| = 4$; *(3 marks)*

(ii) $\arg(z - 1) = -\frac{1}{4}\pi$. *(3 marks)*

(b) Indicate on your sketch the set of points satisfying both

$$|z - 3 + 2i| \leqslant 4$$

and $\arg(z - 1) = -\frac{1}{4}\pi$ *(1 mark)*

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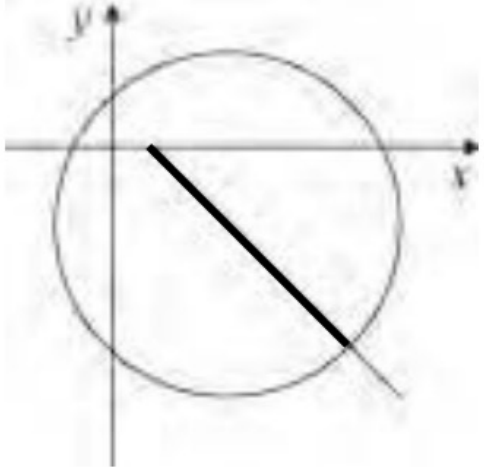
MFP2 (cont)

Q	Solution	Marks	Total	Comments
4				
(a)(i)	Circle	B1		
	Correct centre	B1		
	Enclosing the origin	B1	3	
(ii)	Half line	B1		
	Correct starting point	B1		
	Correct angle	B1	3	
(b)	Correct part of the line **indicated**	B1F	1	
	Total		**7**	

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AQA Jan 2007 FP2

2 (a) Sketch on one diagram:

(i) the locus of points satisfying $|z - 4 + 2\text{i}| = 2$; *(3 marks)*

(ii) the locus of points satisfying $|z| = |z - 3 - 2\text{i}|$. *(3 marks)*

(b) Shade on your sketch the region in which

both $|z - 4 + 2\text{i}| \leqslant 2$

and $|z| \leqslant |z - 3 - 2\text{i}|$ *(2 marks)*

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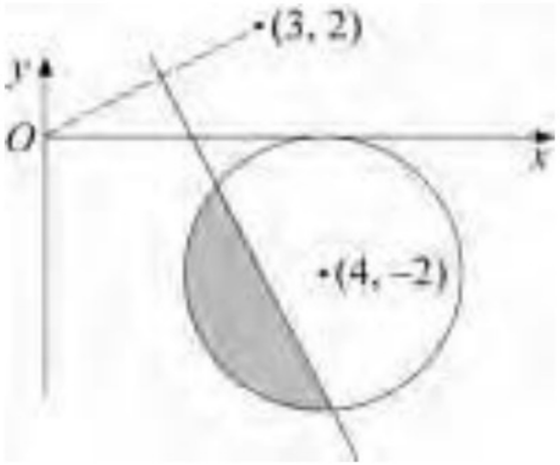
2(a)				
(i)	**Circle** Correct centre Correct radius Touching *x*-axis	 B1 B1 B1	 3	
(ii)	**Line** Point $(3,2)$ indicated Line through $\left(1\frac{1}{2}, 1\right)$ Perpendicular to $(0,0) \rightarrow (3,2)$	 B1 B1√ B1	 3	
(b)	Correct shaded area	B1 B1√	2	For shading inside the circle provided no other area is shaded Must be a circle and a straight line for second B1
	Total		**8**	

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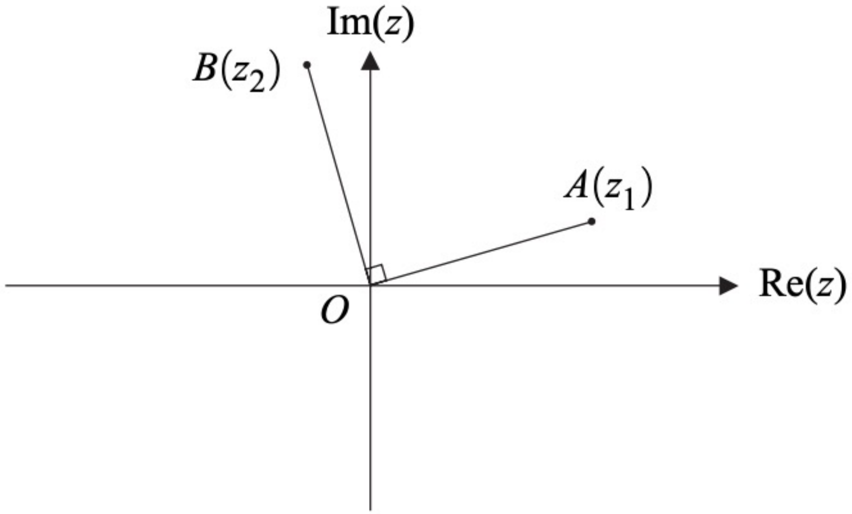
AQA JUNE 2007 FP2

5 The sketch shows an Argand diagram. The points A and B represent the complex numbers z_1 and z_2 respectively. The angle $AOB = 90°$ and $OA = OB$.



(a) Explain why $z_2 = \mathrm{i}z_1$. *(2 marks)*

(b) On a **single** copy of the diagram, draw:

(i) the locus L_1 of points satisfying $|z - z_2| = |z - z_1|$; *(2 marks)*

(ii) the locus L_2 of points satisfying $\arg(z - z_2) = \arg z_1$. *(3 marks)*

(c) Find, in terms of z_1, the complex number representing the point of intersection of L_1 and L_2. *(2 marks)*

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5(a)	Explanation	E2,1,0	2	E1 for $\mathrm{i} = \mathrm{e}^{\frac{\pi \mathrm{i}}{2}}$ or $\mathrm{i}z_1 = -y_1 + \mathrm{i}x_1$
(b)(i)	Perpendicular bisector of *AB*	B1		
	through *O*	B1	2	
(ii)	half-line	B1		If L_2 is taken to be the line *AB* give B0
	from *B*	B1		
	parallel to *OA*	B1	3	
(c)	$(1+\mathrm{i})\,z_1$	M1A1	2	ft if L_2 taken as line *AB*
	Total		**9**	

BACK TO AQA FP2

AQA Jan 2008 FP2

3 A circle C and a half-line L have equations

$$|z - 2\sqrt{3} - \mathrm{i}| = 4$$

and $$\arg(z + \mathrm{i}) = \frac{\pi}{6}$$

respectively.

(a) Show that:

(i) the circle C passes through the point where $z = -\mathrm{i}$; *(2 marks)*

(ii) the half-line L passes through the centre of C. *(3 marks)*

(b) On one Argand diagram, sketch C and L. *(4 marks)*

(c) Shade on your sketch the set of points satisfying both

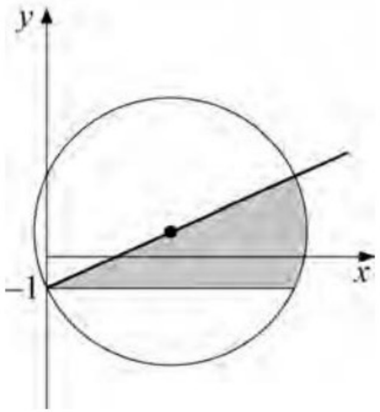
$$|z - 2\sqrt{3} - \mathrm{i}| \leqslant 4$$

and $$0 \leqslant \arg(z + \mathrm{i}) \leqslant \frac{\pi}{6}$$ *(2 marks)*

BACK TO AQA FP2

AQA FP2

MFP2 (cont)

Q	Solution	Marks	Total	Comments
3(a)(i)	$z = -\mathrm{i}$ $\quad \left\|-2\sqrt{3}-2\mathrm{i}\right\| = \sqrt{12+4} = 4$	M1		$\left\|-2\sqrt{3}-2\mathrm{i}\right\|$
		A1	2	4
(ii)	Centre of circle is $2\sqrt{3}+\mathrm{i}$	B1		Do not accept $\left(2\sqrt{3}, 1\right)$ unless attempt to solve using trig
	Substitute into line	M1		
	$\arg\left(2\sqrt{3}+2\mathrm{i}\right) = \frac{\pi}{6}$ shown	A1	3	
(b)				
	Circle: centre correct	B1		
	through $(0,-1)$	B1		
	Half line: through $(0,-1)$	B1		
	through centre of circle	B1	4	
(c)	Shading inside circle and below line	B1F		
	Bounded by $y=-1$	B1	2	
	Total		**11**	

BACK TO AQA FP2

AQA JUNE 2008 FP2

4 (a) A circle C in the Argand diagram has equation

$$|z+5-\mathrm{i}|=\sqrt{2}$$

Write down its radius and the complex number representing its centre. *(2 marks)*

(b) A half-line L in the Argand diagram has equation

$$\arg(z+2\mathrm{i})=\frac{3\pi}{4}$$

Show that $z_1=-4+2\mathrm{i}$ lies on L. *(2 marks)*

(c) (i) Show that $z_1=-4+2\mathrm{i}$ also lies on C. *(1 mark)*

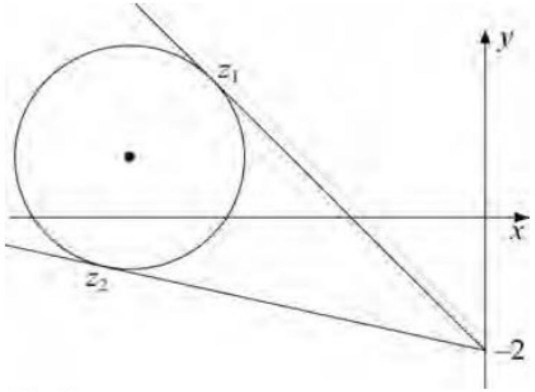
(ii) Hence show that L touches C. *(3 marks)*

(iii) Sketch L and C on one Argand diagram. *(2 marks)*

(d) The complex number z_2 lies on C and is such that $\arg(z_2+2\mathrm{i})$ has as great a value as possible.

Indicate the position of z_2 on your sketch. *(2 marks)*

BACK TO AQA FP2

4(a)	radius $\sqrt{2}$ centre $-5+\text{i}$	B1,B1	2	condone $(-5, 1)$ for centre do not accept $(-5, \text{i})$
(b)	$\arg(z_1+2\text{i}) = \arg(-4+4\text{i})$	M1		
	$= \dfrac{3\pi}{4}$	A1	2	clearly shown eg $\tan^{-1}\left(-\dfrac{1}{1}\right)$
(c)(i)	$\lvert z_1+5-\text{i}\rvert = \lvert 1+\text{i}\rvert = \sqrt{2}$	B1	1	
(ii)	Gradient of line from $(-5, 1)$ to $(-4, 2)$ is 1 $\left(\dfrac{\pi}{4}\right)$	M1A1		M1 for a complete method
	radius $\perp$line $\therefore$ tangent	E1	3	
(iii)				
	Circle correct	B1F		ft incorrect centre or radius
	Half line correct	B1	2	line must touch C generally above the circle
(d)	z_2 in correct place	B1		B0 if z_2 is directly below the centre of C
	with tangent shown	B1	2	
	Total		**12**	

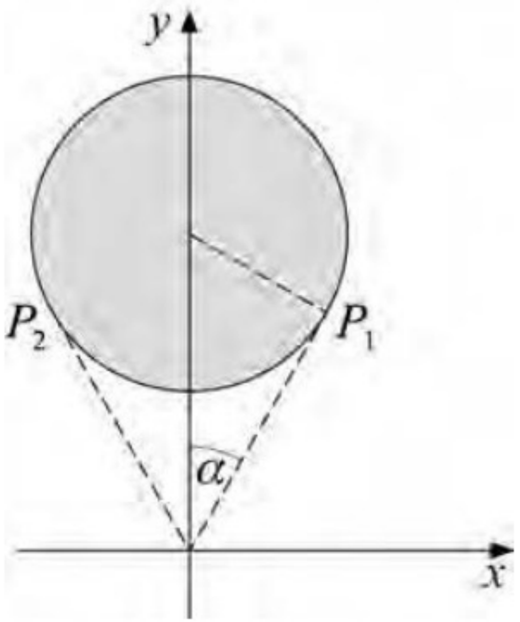
BACK TO AQA FP2

AQA JAN 2009 FP2

2 (a) Indicate on an Argand diagram the region for which $|z - 4\mathrm{i}| \leqslant 2$. *(4 marks)*

(b) The complex number z satisfies $|z - 4\mathrm{i}| \leqslant 2$. Find the range of possible values of $\arg z$. *(4 marks)*

BACK TO AQA FP2

2(a)		B1 B1 B1 B1F	4	Circle Correct centre Correct radius Inside shading
(b)	Correct points P_1 and P_2 indicated	B1F		Possibly by tangents drawn ft mirror image of circle in x-axis
	$\sin\alpha = \frac{2}{4}$	M1		
	$\alpha = \frac{\pi}{6}$	A1		
	Range is $\frac{\pi}{3} \leqslant \arg z \leqslant \frac{2\pi}{3}$	A1	4	Deduct 1 for angles in degrees
	Total		**8**	

BACK TO AQA FP2

AQA JUNE 2009 FP2

6 (a) Two points, A and B, on an Argand diagram are represented by the complex numbers $2+3\mathrm{i}$ and $-4-5\mathrm{i}$ respectively. Given that the points A and B are at the ends of a diameter of a circle C_1, express the equation of C_1 in the form $|z-z_0|=k$.

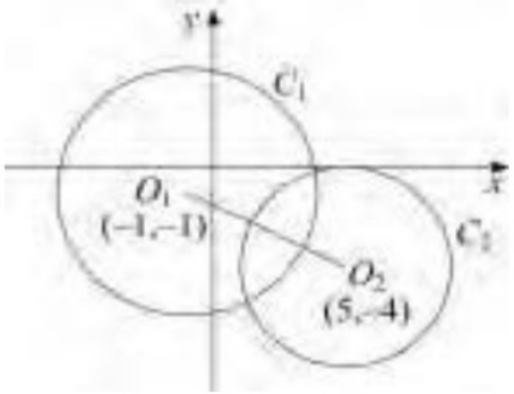
(4 marks)

(b) A second circle, C_2, is represented on the Argand diagram by the equation $|z-5+4\mathrm{i}|=4$. Sketch on one Argand diagram both C_1 and C_2. *(3 marks)*

(c) The points representing the complex numbers z_1 and z_2 lie on C_1 and C_2 respectively and are such that $|z_1-z_2|$ has its maximum value. Find this maximum value, giving your answer in the form $a+b\sqrt{5}$. *(5 marks)*

BACK TO AQA FP2

MFP2 (cont)

Q	Solution	Marks	Total	Comments
6(a)	Centre $-1-\mathrm{i}$ or $(-1,-1)$	B1		
	Radius 5	M1 A1F		ft incorrect centre if used
	$\lvert z+1+\mathrm{i}\rvert=5$ or $\lvert z-(-1-i)\rvert=5$	A1F	4	ft $\lvert z+1+\mathrm{i}\rvert=10$ earns M0B1
(b)				
	C_1 correct centre, correct radius	B1F		ft errors in (a) but fit circles need to intersect and C_1 enclose $(0,0)$
	C_2 correct centre, correct radius	B1		
	Touching x-axis	B1F	3	error in plotting centre
(c)	$O_1O_2=3\sqrt{5}$	M1A1		allow if circles misplaced but O_1O_2 is still $3\sqrt{5}$
	Correct length identified	m1		
	Length is $9+3\sqrt{5}$	M1 A1F	5	ft if r is taken as 10
	Total		**12**	

BACK TO AQA FP2

AQA JAN 2010 FP2

2 (a) On the same Argand diagram, draw:

(i) the locus of points satisfying $|z - 4 + 2\mathrm{i}| = 4$; *(3 marks)*

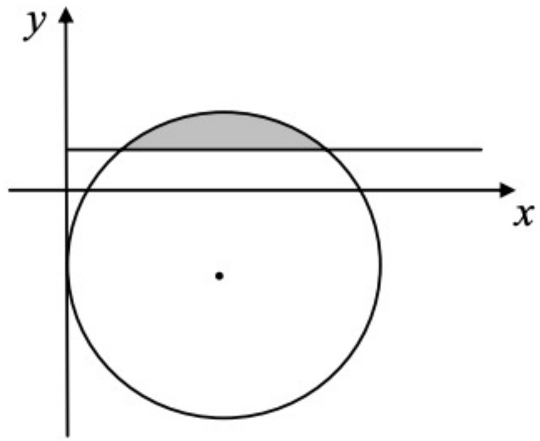
(ii) the locus of points satisfying $|z| = |z - 2\mathrm{i}|$. *(3 marks)*

(b) Indicate on your sketch the set of points satisfying both

$$|z - 4 + 2\mathrm{i}| \leqslant 4$$

and $$|z| \geqslant |z - 2\mathrm{i}|$$ *(2 marks)*

BACK TO AQA FP2

Q	Solution	Marks	Total	Comments
2				
(a)(i)	Circle	B1		
	Correct centre	B1		x-coordinate $\approx -2\times$ y-coordinate in correct quadrant; condone $(4, -2\text{i})$
	Touching y-axis	B1	3	
(ii)	Straight line	B1		
	parallel to x-axis	B1		
	through (0, 1)	B1	3	Assume (0, 1) if distance up y-axis is half distance to top of circle; no other shading outside circle
(b)	Shading: inside circle	B1F		
	above line	B1F	2	Whole question reflected in x-axis loses 2 marks
	Total		**8**	

BACK TO AQA FP2

AQA JUNE 2010 FP2

3 Two loci, L_1 and L_2, in an Argand diagram are given by

$$L_1 : |z + 1 + 3\mathrm{i}| = |z - 5 - 7\mathrm{i}|$$

$$L_2 : \arg z = \frac{\pi}{4}$$

(a) Verify that the point represented by the complex number $2 + 2\mathrm{i}$ is a point of intersection of L_1 and L_2. *(2 marks)*

(b) Sketch L_1 and L_2 on one Argand diagram. *(5 marks)*

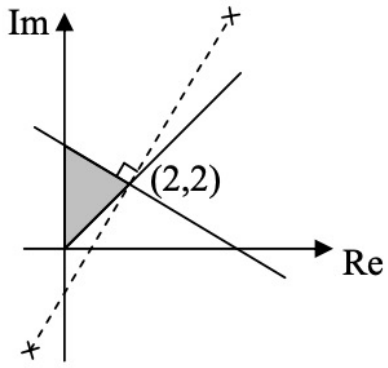
(c) Shade on your Argand diagram the region satisfying

both $$|z + 1 + 3\mathrm{i}| \leqslant |z - 5 - 7\mathrm{i}|$$

and $$\frac{\pi}{4} \leqslant \arg z \leqslant \frac{\pi}{2}$$ *(2 marks)*

BACK TO AQA FP2

MFP2 (cont)

Q	Solution	Marks	Total	Comments
3				
(a)	$\lvert 2+2i+1+3i \rvert = \lvert 2+2i-5-7i \rvert$	B1		Clearly shown do not allow $\lvert 3+5i \rvert = \lvert -3-5i \rvert$ without comment
	$\arg(2+2i) = \frac{\pi}{4}$	B1	2	Clearly shown
(b)	L_1 : straight line with negative gradient perpendicular to line joining	B1		
	$(-1,-3)$ to $(5,7)$	B1		
	through $(2,2)$	B1		The point (2,2) must be shown either by (2,2) or 2+2i or with numbered axes
	L_2 : half line through O	B1		
	through $(2,2)$	B1	5	
(c)	Shading between $\frac{\pi}{4}$ and $\frac{\pi}{2}$	B1		No marks for shading if circles drawn in (b)
	Below L_1	B1	2	
	Total		**9**	

BACK TO AQA FP2

AQA JAN 2011 FP2

1 (a) Sketch on an Argand diagram the locus of points satisfying the equation

$$|z - 4 + 3\mathrm{i}| = 5$$

(3 marks)

(b) (i) Indicate on your diagram the point P representing z_1, where both

$$|z_1 - 4 + 3\mathrm{i}| = 5 \quad \text{and} \quad \arg z_1 = 0$$

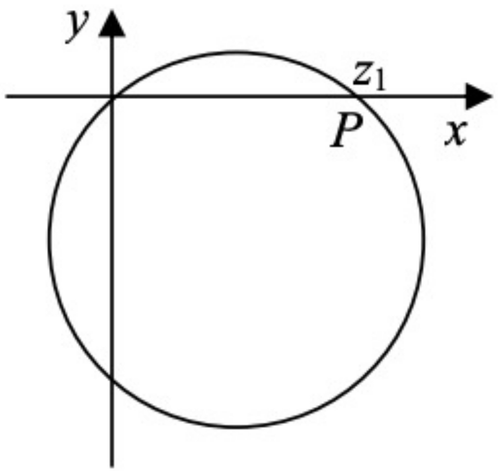
(1 mark)

(ii) Find the value of $|z_1|$.

(1 mark)

BACK TO AQA FP2

MFP2

Q	Solution	Marks	Total	Comments
1(a)	 Circle correct centre through (0, 0)	B1 B1 B1	3	
(b)(i)	z_1 correctly chosen	B1F	1	ft if circle encloses (0, 0)
(ii)	$\|z_1\| = 8$	B1F	1	ft if centre misplotted
	Total		**5**	

BACK TO AQA FP2

AQA JUNE 2011 FP2

1 (a) Draw on the same Argand diagram:

(i) the locus of points for which

$$|z - 2 - 5\mathrm{i}| = 5$$ *(3 marks)*

(ii) the locus of points for which

$$\arg(z + 2\mathrm{i}) = \frac{\pi}{4}$$ *(3 marks)*

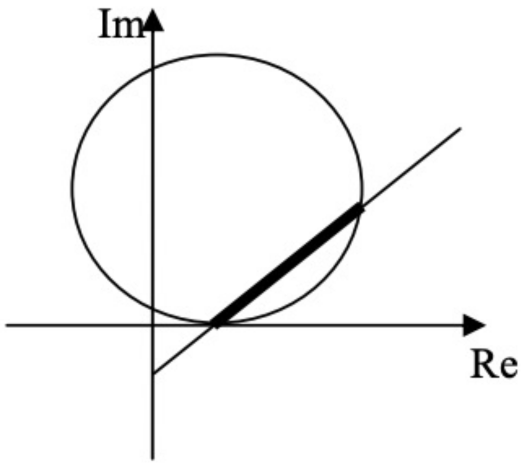
(b) Indicate on your diagram the set of points satisfying both

$$|z - 2 - 5\mathrm{i}| \leqslant 5$$

and $$\arg(z + 2\mathrm{i}) = \frac{\pi}{4}$$ *(2 marks)*

BACK TO AQA FP2

MFP2

Q	Solution	Marks	Total	Comments
1(a)				Use average of whole question if 2 diagrams used
(i)	Circle correct centre touching x-axis	B1 B1 B1F	3	Circle in any position Must be shown ft incorrect centre
(ii)	half-line through (0, –2) through point of contact of circle with x-axis	B1 B1 B1	3	 Can be inferred
(b)	Inside circle On line	B1 B1F	2	 ft errors in position of line and circle
	Total		**8**	

BACK TO AQA FP2

AQA JAN 2012 FP2

2 (a) Draw on an Argand diagram the locus L of points satisfying the equation $\arg z = \frac{\pi}{6}$.

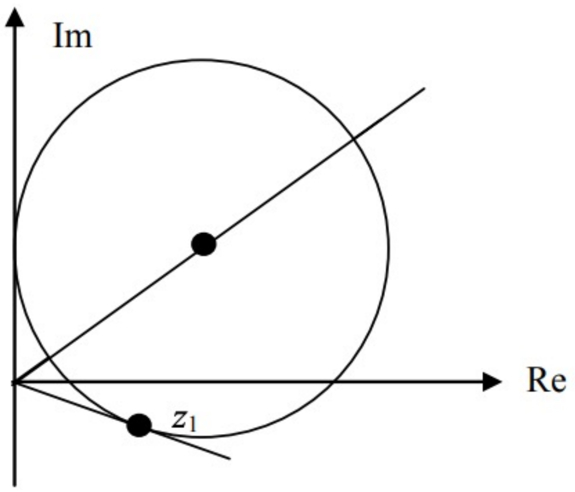
(1 mark)

(b) (i) A circle C, of radius 6, has its centre lying on L and touches the line $\text{Re}(z) = 0$. Draw C on your Argand diagram from part **(a)**. *(2 marks)*

(ii) Find the equation of C, giving your answer in the form $|z - z_0| = k$. *(3 marks)*

(iii) The complex number z_1 lies on C and is such that $\arg z_1$ has its least possible value. Find $\arg z_1$, giving your answer in the form $p\pi$, where $-1 < p \leqslant 1$. *(2 marks)*

BACK TO AQA FP2

Q	Solution	Marks	Total	Comments
2(a)				
	Half-line with gradient < 1	B1	1	condone a short line, ie it stops at or inside circle
(b)(i)	Circle centre on L, x-coord 6 indicated touching $\text{Re } z = 0$ not at (0, 0)	B1 B1	 2	 not touching Re axis
(ii)	y-coord of centre is $2\sqrt{3}$ or $\frac{6}{\sqrt{3}}$	B1		OE; PI
	$z_0 = 6 + 2\sqrt{3}\,\text{i}$, $k = 6$	B1F, B1	 3	ft error in coords of centre
(iii)	Point z_1 shown	B1		PI
	$\arg \pi z_1 = -\frac{1}{6}$	B1	2	
	Total		**8**	

BACK TO AQA FP2

AQA JUNE 2012 FP2

2 (a) Draw on the Argand diagram below:

(i) the locus of points for which

$$|z - 2 - 3i| = 2$$ *(3 marks)*

(ii) the locus of points for which

$$|z + 2 - i| = |z - 2|$$ *(3 marks)*

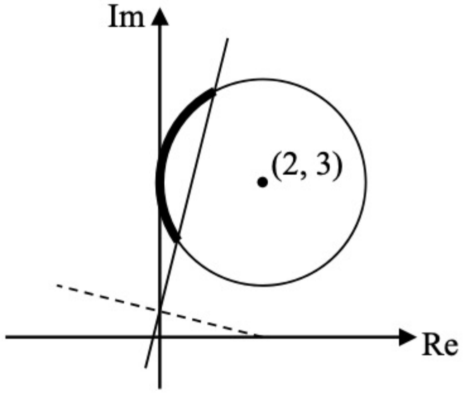
(b) Indicate on your diagram the points satisfying both

$$|z - 2 - 3i| = 2$$

and $$|z + 2 - i| \leqslant |z - 2|$$ *(1 mark)*

BACK TO AQA FP2

MFP2

Q	Solution	Marks	Total	Comments
2(a)				
(i)	Circle Correct centre Touching Im axis	B1 B1 B1	3	Convex loop Some indication of position of centre
(ii)	Straight line well to left of centre	B1		$\frac{1}{2}$ line through $(0, \frac{1}{2})$ B0
	through $(0, \frac{1}{2})$	B1		Point approximately between 0 and 1
	$\perp$ to line joining $(-2,1)$ and $(2,0)$ NB 0/3 for line parallel to x-axis 0/3 for line joining the two points $(-2, 1)$ and $(2,0)$ 0/3 for line joining (0,0) to centre of circle	B1	3	
(b)	Minor arc indicated	B1F	1	ft incorrect position of line or circle
	Total		**7**	

BACK TO AQA FP2

AQA JAN 2013 FP2

2 Two loci, L_1 and L_2, in an Argand diagram are given by

$$L_1 : |z + 6 - 5\mathrm{i}| = 4\sqrt{2}$$

$$L_2 : \quad \arg(z + \mathrm{i}) = \frac{3\pi}{4}$$

The point P represents the complex number $-2 + \mathrm{i}$.

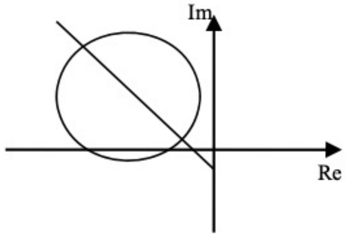
(a) Verify that the point P is a point of intersection of L_1 and L_2. *(2 marks)*

(b) Sketch L_1 and L_2 on one Argand diagram. *(6 marks)*

(c) The point Q is also a point of intersection of L_1 and L_2. Find the complex number that is represented by Q. *(2 marks)*

BACK TO AQA FP2

MFP2 (cont)

Q	Solution	Marks	Total	Comments
2(a)	$\lvert 4-4i\rvert = \sqrt{16+16} = \sqrt{32} = 4\sqrt{2}$	B1		verification that $\lvert -2+i+6-5i\rvert = 4\sqrt{2}$
	$\arg(-2+2i) = \pi - \tan^{-1}(1) = \frac{3\pi}{4}$	B1	2	verification that $\arg(z+i) = \frac{3\pi}{4}$
				
(b)	Circle	M1		freehand circle sketched
	Centre at $-6+5i$	A1		clear from diagram or centre stated
	Cutting Re axis but not cutting Im axis	A1		
	"Straight" line	M1		freehand line
	Half line from $0-i$	A1		not horizontal or vertical but end point at $0-i$ must be clear from diagram/stated
	gradient -1 (approx)	A1	6	making $45°$ to negative Re axis and positive Im axis
(c)	Calculation based on fact that L_2 passes through centre of L_1	M1		idea of vector $\begin{bmatrix}-4\\4\end{bmatrix}$ from centre
	Q represents $-10+9i$	A1	2	must write as a complex number
	Total		**10**	

BACK TO AQA FP2

AQA JUNE 2013 FP2

1 (a) Sketch on an Argand diagram the locus of points satisfying the equation

$$|z - 6\mathrm{i}| = 3$$

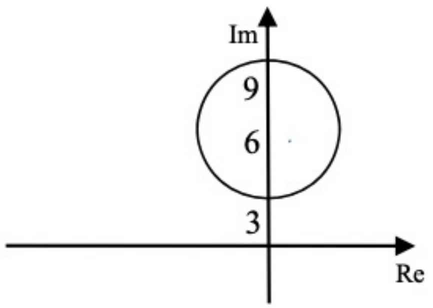
(3 marks)

(b) It is given that z satisfies the equation $|z - 6\mathrm{i}| = 3$.

(i) Write down the greatest possible value of $|z|$. *(1 mark)*

(ii) Find the greatest possible value of $\arg z$, giving your answer in the form $p\pi$, where $-1 < p \leqslant 1$.
(3 marks)

BACK TO AQA FP2

Q	Solution	Marks	Total	Comments
1(a)				
	Circle	M1		freehand circle
	Centre at 6i	A1		6 marked on Im axis as centre
	Radius 3 & cutting positive Im axis twice	A1	3	radius of 3 clearly indicated with circle in position shown
(b)(i)	(Max $\lvert z \rvert$ is) 9	B1	1	
(ii)	Tangent from O to circle	M1		FT their circle position
	Angle of $\frac{\pi}{6}$ or $\frac{\pi}{3}$ ***correctly*** marked	A1		PI ; condone degrees for first A1
	(Max arg z is) $\frac{2\pi}{3}$	A1cso	3	exactly this
	Total		**7**	

BACK TO AQA FP2