1.

Q3	Scheme	Marks	AOs
(a)	$H: xy = 9 \quad l_1: y = mx + c$ $x(mx + c) = 9 \Rightarrow mx^2 + cx - 9 = 0$ discriminant $= 0 \Rightarrow c^2 - 4m(-9) = 0$	M1	3.1a
	$c^2 + 36m = 0$	A1	2.1
		(2)	
(b)	$C: y^2 = 32x \Rightarrow a = 8 \Rightarrow Q$ is $(-8, 0)$	B1	2.2a
	Substitute into $l_1 \Rightarrow 0 = -8m + c \Rightarrow c = 8m$ or $m = \frac{c}{8}$ $(8m)^2 + 36m = 0$ or $c^2 + \frac{9}{2}c = 0$	M1	1.1b
	$64m^2 + 36m = 0 \Rightarrow 4m(16m + 9) = 0 \Rightarrow m = -\frac{9}{16}$ or $c = -\frac{9}{2} \Rightarrow m = -\frac{9}{16}$	A1	1.1b
	Gradient of $l_2 = -\frac{1}{m} = \frac{16}{9}$ or exact equivalents	A1ft	2.2a
		(4)	
	Alternative (using equation of tangent)		
	$C: y^2 = 32x \Rightarrow a = 8 \Rightarrow Q$ is $(-8, 0)$	B1	2.2a
	$xy = 9 \Rightarrow \frac{dy}{dx} = -\frac{9}{x^2}$ or $x\frac{dy}{dx} + y = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x}$ and $x = 3t, y = \frac{3}{t} \Rightarrow \frac{dy}{dx} = -\frac{1}{t^2}$ OR $\frac{dx}{dt} = 3, \frac{dy}{dt} = -\frac{3}{t^2} \Rightarrow \frac{dy}{dx} = -\frac{1}{t^2}$ l_1 is $y - \frac{3}{t} = -\frac{1}{t^2}(x - 3t)$ or $x + 2t^2y = 6t$ $(-8, 0) \Rightarrow -\frac{3}{t} = -\frac{1}{t^2}(-8 - 3t)$ or $-8 = 6t$	M1	1.1b
	$3t = -8 - 3t \Rightarrow 6t = -8 \Rightarrow t = -\frac{4}{3}$ So P is $(-4, -\frac{9}{4})$ and gradient PQ $= \frac{-\frac{9}{4} - 0}{-4 - (-8)} = \frac{-\frac{9}{4}}{4} = -\frac{9}{16}$	A1	1.1b
	Gradient of l_2 is $-\frac{1}{m} = \frac{16}{9}$ or exact equivalents	A1ft	2.2a
		(4)	
			(6 marks)

2.

Q2	Scheme	Marks	AOs
(a)	$2\frac{d^2y}{dt^2}+y\frac{dy}{dt}-5=0$ $2\left(\frac{d^2y}{dt^2}\right)_0+5(0.72)-5=0 \Rightarrow \left(\frac{d^2y}{dt^2}\right)_0=\ldots$	M1	3.4
	$\left(\frac{d^2y}{dt^2}\right)_0=\frac{5-5(0.72)}{2}=0.7$ (km min^{-2})	A1	1.1b
		(2)	
(b)	$\left(\frac{d^2y}{dt^2}\right)_0 \approx \frac{y_1-2y_0+y_{-1}}{h^2} \Rightarrow 0.7 \approx \frac{y_1-2(5)+y_{-1}}{0.25^2}$	M1	3.4
	$\left(\frac{dy}{dt}\right)_0 \approx \frac{y_1-y_{-1}}{2h} \Rightarrow 0.72 \approx \frac{y_1-y_{-1}}{2(0.25)}$	M1	3.4
	$10.04375 \approx y_1+y_{-1}$ $\quad 0.36 \approx y_1-y_{-1}$ $10.40375 \approx 2y_1$ $y_1 \approx 5.201875$	M1	2.2a
	5.2 (km)	A1	1.1b
		(4)	
			(6 marks)

Notes

(a)

M1: Uses the model by substituting $y_0 = 5$ and $\left(\frac{dy}{dt}\right)_0 = 0.72$ into the given differential equation and obtains a value for $\left(\frac{d^2y}{dt^2}\right)_0$

A1: 0.7 oe

(b)

M1: Uses approximation formula with $y_0 = 5$, their $\left(\frac{dy}{dt}\right)_0$ and their h to obtain an equation in y_1 and y_{-1}

M1: Uses approximation formula with $\left(\frac{dy}{dt}\right)_0$ and their h to obtain an equation in y_1 and y_{-1}

M1: Solves their simultaneous equations to obtain a value for y_1

A1: 5200 m or 5.2 (km)

3.

Question Number	Scheme	Marks
3		
	$x^2+x-2<\frac{1}{2}x+\frac{5}{2}$	
	$2x^2+x-9<0$	M1
	CVs $x=\frac{-1\pm\sqrt{73}}{4}$	A1
	$-x^2-x+2<\frac{1}{2}x+\frac{5}{2}$	M1
	$2x^2+3x+1>0 \quad (2x+1)(x+1)>0$	M1
	CVs $x=-\frac{1}{2},\ -1$	A1
	$\frac{-1-\sqrt{73}}{4}<x<-1, \quad -\frac{1}{2}<x<\frac{-1+\sqrt{73}}{4}$	M1A1
		[7]
	No algebra implies no marks	
NB	The first 5 marks can all be awarded if equations rather than inequalities are shown	
M1	Obtain and solve a 3TQ (any valid method including calculator)	
A1	2 correct CVs Allow decimal equivalents (1.886..., -2.386...), min 3 sf, rounded or truncated	
M1	Multiply either side by -1	
M1	Obtain and solve a 3TQ (any valid method including calculator)	
A1	2 correct CVs	
M1	Form 2 double inequalities with their CVs. No overlap between these inequalities. Correct inequality signs required here or for final mark	
A1	Correct inequalities obtained. Values must be exact, but note that 0.5 is exact. Allow "and" but not "$\cap$". May be written in set language with "$\cup$" and round brackets	

4.

Question Number	Scheme	Marks
1(a)	$\frac{d^3y}{dx^3}+3\frac{dy}{dx}+3x\frac{d^2y}{dx^2}=-2\sin x$	M1M1
	$\frac{d^3y}{dx^3}=-2\sin x-3\frac{dy}{dx}-3x\frac{d^2y}{dx^2}$	A1 (3)
(b)	$\frac{d^3y}{dx^3}=-3\times 5=-15$	B1 (1)
(c)	$\frac{d^2y}{dx^2}=-3\times 0\times 5+2=2$	B1
	$y=2+5x+x^2-\frac{5}{2}x^3$	M1A1 (3)
		[7]

(a)	Accept the dashed notation throughout this question.
M1	Differentiate $3x\frac{dy}{dx}$ with respect to x. The product rule must be used for $x\frac{dy}{dx}$ with at least one term correct
M1	Differentiate $\frac{d^2y}{dx^2}$ and $2\cos x$. $\frac{d^2y}{dx^2}\to\frac{d^3y}{dx^3}$ $2\cos x\to\pm 2\sin x$
A1	$\frac{d^3y}{dx^3}=-3\left(x\frac{d^2y}{dx^2}+\frac{dy}{dx}\right)-2\sin x$. Give A0 if not rearranged to have $\frac{d^3y}{dx^3}=\ldots$
(b)	
B1	$\frac{d^3y}{dx^3}=-15$ provided 3 terms in result in (a)
(c)	
B1	$\frac{d^2y}{dx^2}=2$ can be implied by a correct x^2 term in the expansion
M1	Use of a correct Taylor expansion with their values for $\frac{d^3y}{dx^3}$ and $\frac{d^2y}{dx^2}$ 2! or 2, 3! or 6·
A1	$y=2+5x+x^2-\frac{5}{2}x^3$ Must include $y=\ldots$ or f(x) = .. provided f(x) has been defined to be y somewhere in the work.

5.

Question Number	Scheme	Marks
4 (a)	$y^2 = z^{-1} \Rightarrow 2y\frac{dy}{dx} = -\frac{1}{z^2}\frac{dz}{dx}$ oe eg $\frac{dy}{dx} = -\frac{1}{2}z^{-\frac{3}{2}}\frac{dz}{dx}$	B1
	$2y\frac{dy}{dx} + 4y^2 = 6xy^4$	
	$-\frac{1}{z^2}\frac{dz}{dx} + \frac{4}{z} = \frac{6x}{z^2}$	M1
	$\frac{dz}{dx} - 4z = -6x$ *	A1 * (3)
(b)	$\text{IF} = e^{\int -4dx} = e^{-4x}$	B1
	$e^{-4x}\left(\frac{dz}{dx} - 4z\right) = e^{-4x} \times -6x$	
	$ze^{-4x} = -6\int xe^{-4x}dx$	M1
	$= -6\left[-\frac{1}{4}xe^{-4x} + \int \frac{1}{4}e^{-4x}dx\right]$	M1
	$= -6\left[-\frac{1}{4}xe^{-4x} - \frac{1}{16}e^{-4x}\right] (+c)$ oe	A1
	$= \frac{3}{2}xe^{-4x} + \frac{3}{8}e^{-4x} (+c)$	
	$z = \frac{3}{2}x + \frac{3}{8} + ce^{4x}$ oe	A1 (5)
ALT	$\frac{dz}{dx} - 4z = -6x$	
	$m - 4 = 0 \Rightarrow m = 4 \Rightarrow$ CF is $z = Ae^{4x}$	B1
	PI: $z = \lambda + \mu x$	M1
	$\frac{dz}{dx} = \mu \Rightarrow \mu - 4(\lambda + \mu x) = -6x$	
	$4\mu = 6$ $4\lambda = \mu, \Rightarrow \mu = \frac{3}{2}, \lambda = \frac{3}{8}$	M1,A1
	$z = \frac{3}{2}x + \frac{3}{8} + Ae^{4x}$	A1

Question Number	Scheme	Marks
(a)		
B1	Correct derivative seen explicitly or used	
M1	Substitutions made. Only award when an equation in x and z only is reached (if working equation I to II) or an equation in x and y is reached (if working II to I)	
A1 *	Correct result obtained with no errors in working	
(b)		
B1	Correct IF seen explicitly or used	
M1	Multiply through by their IF and integrate the LHS. Accept I for e^{-4x} on LHS only	
M1	Apply parts in the correct direction to RHS to obtain $Axe^{-4x} + B\int e^{-4x}dx$ with $A = \pm\frac{3}{2}$ and $B = \pm\frac{3}{2}$	
A1	Correct integration of RHS, constant not needed	
A1	Include the constant and treat it correctly. Answer in form $z = \ldots$	
ALT		
B1	Correct CF May not be seen until GS is formed	
M1	For a PI of the correct form	
M1	Differentiate their PI, substitute in the equation and extract 2 equations for the unknowns	
A1	Solve the two equations to obtain correct values for the unknowns	
A1	Correct GS obtained	

6.

Question	Scheme		Marks
6(a)	$\overrightarrow{AB}=\begin{pmatrix}-2\\1\\1\end{pmatrix}, \overrightarrow{AC}=\begin{pmatrix}1\\-1\\3\end{pmatrix}, \overrightarrow{BC}=\begin{pmatrix}3\\-2\\2\end{pmatrix}$	Two correct vectors in Π Can be negatives of those shown	B1
	$\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\-2 & 1 & 1\\1 & -1 & 3\end{vmatrix}=\begin{pmatrix}4\\7\\1\end{pmatrix}$	M1: Attempt cross product of two vectors lying in Π (At least one no. to be correct.) A1: Correct normal vector	M1 A1
	$\begin{pmatrix}4\\7\\1\end{pmatrix}\bullet\begin{pmatrix}1\\2\\3\end{pmatrix}=4+14+3$	Attempt scalar product with their normal and a point in the plane	dM1
	$4x+7y+z=21$	Cao (oe)	A1
			(5)
	Alternative 1		
	$a+2b+3c=d$ $-a+3b+4c=d$ $2a+b+6c=d$	Correct equations	B1
	$a=\frac{4}{21}d,\ b=\frac{1}{3}d,\ c=\frac{1}{21}d$	M1: Solve for a, b and c in terms of d A1: Correct equations	M1 A1
	$d=21\Rightarrow a=...,\ b=...,\ c=...$	Obtains values for a, b, c and d	M1
	$4x+7y+z=21$	Cao (oe)	A1
			(5)
	Alternative 2: Using $\mathbf{r}=\mathbf{a}+s\mathbf{b}+t\mathbf{c}$ where b and c are vectors in Π		
	Two correct vectors in the plane	See main scheme	B1
	Eg $\mathbf{r}=\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}1\\2\\3\end{pmatrix}+s\begin{pmatrix}-2\\1\\1\end{pmatrix}+t\begin{pmatrix}1\\-1\\3\end{pmatrix}$		M1
	$x=1-2s+t$ $y=2+s-t$ $z=3+s+3t$	Deduce 3 correct equations	A1
	$4x+7y+z=21$	M1: Eliminate s, t A1: Cao	M1 A1
			(5)

Question	Scheme		Marks
6(b)	AD•AB×AC	Attempt suitable triple product	M1
	$=\begin{pmatrix}4\\7\\1\end{pmatrix}\bullet\begin{pmatrix}k-1\\2\\11\end{pmatrix}=4k-4+14+11$		
	$\therefore\frac{1}{6}(4k+21)=6$	M1: Set $\frac{1}{6}$(their triple product) = 6 A1: Correct equation	dM1 A1
	$k=\frac{15}{4}$	Cao (oe)	A1
			(4)
	Alternative		
	Area ABC $=\frac{1}{2}\lvert\overline{AB}\rvert\lvert\overline{AC}\rvert=\frac{1}{2}\sqrt{6}\sqrt{11}$ D to Π is $\frac{4k+28+14-21}{\sqrt{16+49+1}}$	Attempt area *ABC* and distance between *D*and*Π*	M1
	$\frac{1}{6}\sqrt{6}\sqrt{11}\frac{4k+28+14-21}{\sqrt{16+49+1}}=6$	M1: Set $\frac{1}{3}$(their area x their distance) = 6 A1: Correct equation	dM1 A1
	$k=\frac{15}{4}$	Cao (oe)	A1
			(4)
			(9 marks)

(c)	$a=4\Rightarrow \mathbf{A}^{-1}=\frac{1}{6}\begin{pmatrix}2&6&-2\\-3&3&0\\1&-3&2\end{pmatrix}$	Correct inverse (follow through their matrix from (a))	B1ft
	$=\frac{1}{6}\begin{pmatrix}2&6&-2\\-3&3&0\\1&-3&2\end{pmatrix}\begin{pmatrix}12-6\lambda\\4+2\lambda\\6+3\lambda\end{pmatrix}=\ldots$	Attempt to multiply the parametric form of l_2 by their inverse	M1
	$=\begin{pmatrix}6-\lambda\\-4+4\lambda\\2-\lambda\end{pmatrix}$	Correct parametric form	A1
	$\mathbf{r}=\begin{pmatrix}6\\-4\\2\end{pmatrix}+\lambda\begin{pmatrix}-1\\4\\-1\end{pmatrix}$	Correct equation (allow equivalent forms) but if given as $l=\ldots$ award A0	A1
			(4)

7.

Question Number	Scheme	Marks
(a)	$\frac{2x}{a^2}+\frac{2y}{b^2}\frac{dy}{dx}=0$ and so $\frac{dy}{dx}=-\frac{xb^2}{ya^2}=-\frac{b\cos\theta}{a\sin\theta}$	M1 A1
	$\therefore\ y-b\sin\theta=-\frac{b\cos\theta}{a\sin\theta}(x-a\cos\theta)$	M1
	Uses $\cos^2\theta+\sin^2\theta=1$ to give $\frac{x\cos\theta}{a}+\frac{y\sin\theta}{b}=1$ *	A1cso (4)
(b)	Gradient of circle is $-\frac{\cos\theta}{\sin\theta}$ and equation of tangent is	M1
	$y-a\sin\theta=-\frac{\cos\theta}{\sin\theta}(x-a\cos\theta)$ or sets $a=b$ in previous answer	
	So $y\sin\theta+x\cos\theta=a$	A1 (2)
(c)	Eliminate x or y to give $y\sin\theta(\frac{a}{b}-1)=0$ or $x\cos\theta(\frac{b}{a}-1)=b-a$	M1
	l_1 and l_2 meet at $(\frac{a}{\cos\theta},0)$	A1, B1 (3)
(d)	The locus of R is part of the line $y=0$, such that $x\geq a$ and $x\leq -a$ Or clearly labelled sketch. Accept "real axis"	B1, B1 (2) (11 marks)

Notes

a1M1: Finding gradient in terms of θ. Must use calculus.
a1A1: cao
a2M1: Finding equation of tangent
a2A1: cso (answer given). Need to get $\cos^2\theta+\sin^2\theta$ on the same side.
b1M1: Finding gradient and equation of tangent, **or** setting $a=b$.
b1A1: cao need not be simplified.
c1M1: As scheme
c1A1: $x=\frac{a}{\cos\theta}$, need not be simplified.
c1B1: $y=0$, need not be simplified.
d1B1: Identifying locus as $y=0$ or real/'x' axis.
d2B1: Depends on previous B mark, identifies correct parts of $y=0$. Condone use of strict inequalities.

8.

Question	Scheme	Marks	AOs
	$u = x^3 \Rightarrow \frac{du}{dx} = 3x^2, \frac{d^2u}{dx^2} = 6x, \frac{d^3u}{dx^3} = 6$	M1	1.1b
	$v = \sin kx \Rightarrow \frac{dv}{dx} = k\cos kx, \frac{d^2v}{dx^2} = -k^2 \sin kx, \frac{d^3v}{dx^3} = -k^3 \cos kx,$ $\frac{d^4v}{dx^4} = k^4 \sin kx, \frac{d^5v}{dx^5} = k^5 \cos kx$	M1	2.1
	$\frac{d^5y}{dx^5} = x^3k^5\cos kx + 5\times 3x^2 \times k^4 \sin kx + \frac{5\times 4}{2}\times 6x \times \left(-k^3 \cos kx\right) +$ $\frac{5\times 4\times 3}{3!}\times 6\times\left(-k^2 \sin kx\right)$	M1	2.1
	$= \left(k^2x^2 - 60\right)k^3 x\cos kx + 15\left(k^2x^2 - 4\right)k^2 \sin kx$	A1	1.1b
		(4)	
			(4 marks)

Notes

M1: Differentiates $u = x^3$ three times. Need to see $x^3 \to ...x^2 \to ..x \to k$

M1: Uses $v = \sin kx$ to establish the form of the derivatives. Need to see at least alternating $k\cdots\sin kx$ and $k\cdots\cos kx$ with increasing powers of k for at least 3 derivatives.

M1: Uses a correct formula with 2 and 3! (or 6) with terms shown to disappear after the fourth term. This needs to be a correct application of the theorem so that the correct binomial coefficients need to go with the correct pairings of their derivatives. If there is any doubt, at least 3 terms should have the correct structure. Allow equivalent notation for the binomial coefficients e.g. $\begin{pmatrix}5\\0\end{pmatrix}, \begin{pmatrix}5\\1\end{pmatrix}$ etc. or ${}^5C_0, {}^5C_1$ etc.

A1: Correct expression in the required form with correct values of A, B and C. Apply isw if necessary e.g. if a correct expression is followed by $A = 60$, $B = 15$, $C = -4$
(NB $A = -60$, $B = 15$, $C = -4$)

If there is no use of Lebnitz's theorem e.g. repeated differentiation of products, this scores no marks.

9.

Q5	Scheme	Marks	AOs
(a)	As $x \to 5$, $-4+\sqrt{1+3x} \to (-4+\sqrt{1+3(5)}=)\ 0$ and $x^4-24x^2-25 \to (625-600-25=)\ 0$ which leads to the indeterminate form of $\frac{0}{0}$	B1	2.1
		(1)	
(b)	Letting $f(x)=\sqrt{1+3x}=(1+3x)^{\frac{1}{2}}$ $f'(x)=\frac{1}{2}(1+3x)^{-\frac{1}{2}}(3)=\frac{3}{2\sqrt{1+3x}}$ $\quad f''(x)=-\frac{1}{2}\left(\frac{3}{2}\right)(1+3x)^{-\frac{3}{2}}(3)=\frac{-9}{4\left(\sqrt{1+3x}\right)^3}$	M1 A1	1.1b 1.1b
	$f(5)=\sqrt{16}=4$ $\quad f'(5)=\frac{3}{2\sqrt{16}}=\frac{3}{8}$ $\quad f''(5)=\frac{-9}{4\left(\sqrt{16}\right)^3}=-\frac{9}{256}$	M1	1.1b
	$f(x)=f(a)+(x-a)f'(a)+\frac{(x-a)^2}{2!}f''(a)+\ldots=4+\frac{3}{8}(x-5)-\frac{9}{512}(x-5)^2+\ldots$	M1 A1	2.5 1.1b
		(5)	
(c)	$\frac{-4+\sqrt{1+3x}}{x^4-24x^2-25}=\frac{-4+4+\frac{3}{8}(x-5)-\frac{9}{512}(x-5)^2+\ldots}{x^4-24x^2-25}$	M1	2.2a
	$x^4-24x^2-25=(x^2-25)(x^2+1)$ or $(x-5)(x+5)(x^2+1)$	B1	1.1b
	$\frac{-4+\sqrt{1+3x}}{x^4-24x^2-25}=\frac{\frac{3}{8}(x-5)-\frac{9}{512}(x-5)^2+\ldots}{(x-5)(x+5)(x^2+1)}=\frac{\frac{3}{8}-\frac{9}{512}(x-5)+\ldots}{(x+5)(x^2+1)}$	M1	2.2a
	So $\lim_{x\to 5}\left(\frac{-4+\sqrt{1+3x}}{x^4-24x^2-25}\right)=\frac{\frac{3}{8}}{10(5^2+1)}=\frac{3}{2080}$	A1	1.1b
		(4)	
(d)	$g'(x)=4x^3-48x$	B1	1.1b
	$\lim_{x\to 5}\left(\frac{-4+\sqrt{1+3x}}{x^4-24x^2-25}\right)=\lim_{x\to 5}\left(\frac{\frac{3}{2\sqrt{1+3x}}}{4x^3-48x^2}\right)$	M1	1.1b
	$=\frac{\frac{3}{8}}{260}=\frac{3}{2080}$	A1ft	2.1
		(3)	
			(13 marks)