Pearson Edexcel

Mark Scheme (Results)

October 2021

Pearson Edexcel GCE Mathematics

Pure 1 Paper 9MA0/01

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the first candidate in exactly the same way as they mark the last.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- There is no ceiling on achievement. All marks on the mark scheme should be used appropriately.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification may be limited.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, the team leader must be consulted.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 100.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for ‘knowing a method and attempting to apply it’, unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- $\ast$ The answer is printed on the paper
- The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses <u>and indicates which response they wish to submit</u>, examiners should mark this response.
   If there are several attempts at a question <u>which have not been crossed out</u>, examiners should mark the final answer which is the answer that is the <u>most complete</u>.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

General Principles for Further Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

Method mark for solving 3 term quadratic:

1. Factorisation

$(x^2+bx+c)=(x+p)(x+q),\ \text{where}\ |pq|=|c|$, leading to $x=\ldots$

$(ax^2+bx+c)=(mx+p)(nx+q),\ \text{where}\ |pq|=|c|\ \text{and}\ |mn|=|a|$, leading to $x=\ldots$

2. Formula

Attempt to use the correct formula (with values for *a*, *b* and *c*)

3. Completing the square

Solving $x^2+bx+c=0$: $\left(x \pm \frac{b}{2}\right)^2 \pm q \pm c = 0,\quad q \neq 0$, leading to $x=\ldots$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n \to x^{n-1})$

2. Integration

Power of at least one term increased by 1. $(x^n \to x^{n+1})$

Use of a formula

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

Method mark for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is not quoted, the method mark can be gained by implication from correct working with values but may be lost if there is any mistake in the working.

Exact answers

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

Question	Scheme	Marks	AOs
1	Let the consecutive odd integers be $2n-1$ and $2n+1$ $(2n-1)^2+(2n+1)^2=4n^2-4n+1+4n^2+4n+1=\ldots$	M1	2.1
	$=8n^2+2$	A1	1.1b
	So $(2n-1)^2+(2n+1)^2$ is always 2 more than a multiple of 8	A1	2.4
		(3)	
			(3 marks)

Notes

M1: Starts the proof by stating 2 consecutive odd numbers, squares and adds and collects terms
A1: Correct expression
A1: Completes the proof with no errors and an appropriate conclusion

2(a)	$f(2)=3(2)^3-7(2)^2+7(2)-10=...$	M1	1.1b
	$f(2)=38-38=0\Rightarrow(x-2)$ is a factor of $f(x)$ *	A1*	2.1
		(2)	
(b)	$a=3$ or $c=5$	B1	2.2a
	$f(x)=(x-2)(...x^2+...x+...)$	M1	1.1b
	$a=3,\ b=-1,\ c=5$	A1	1.1b
		(3)	
(c)	$3x^2-x+5=0\Rightarrow b^2-4ac=(-1)^2-4(3)(5)=...$ or e.g. $3x^2-x+5=0\Rightarrow 3\left(x^2-\frac{1}{3}x+\frac{5}{3}\right)=0\Rightarrow\left(x-\frac{1}{6}\right)^2-\frac{1}{36}+\frac{5}{3}\Rightarrow\left(x-\frac{1}{6}\right)^2=...$ or e.g. $\frac{dy}{dx}=6x-1=0\Rightarrow x=...,\Rightarrow y=...$	M1	3.1a
	$(-1)^2-4(3)(5)=-59\Rightarrow b^2-4ac<0$ or e.g. $\left(x-\frac{1}{6}\right)^2=-\frac{59}{36}$ and square numbers cannot be negative or e.g. $\frac{dy}{dx}=0\Rightarrow y=\frac{59}{12}$ so the minimum is above the x-axis So the quadratic has no real roots and so $f(x) = 0$ has only 1 real root	A1	2.4
		(2)	
			(7 marks)

Notes

(a)
M1: Attempts $f(2)$
A1*: Clearly shows $f(2) = 0$ and makes a suitable conclusion
(b)
B1: Deduces the correct value of a or c
M1: Complete method to obtain values for a, b and c. May use inspection or expand to give $ax^3+(b-2a)x^2+(c-2b)x-2c$ and compare coefficients.
A1: All correct stated or embedded
(c)
M1: Starts the process of showing that their 3-term quadratic has no real roots. E.g. considers discriminant or attempts to solve by completing the square or differentiates to find turning point
A1: Fully correct work with appropriate conclusion for their chosen method

Question Number	Scheme	Marks
3(a)	$y = \mathrm{f}(x)+3$ $y = \mathrm{f}(2x)$ $(9,6)$ $(4.5,3)$ $P(9,3)$	
	One correct sketch drawn and labelled correctly	M1
	One correct sketch drawn and labelled and with correct point	A1
	Completely correct sketches with both points	A1
		(3)
(b)	Sets $\sqrt{x}+3=\sqrt{2x}$	B1
	$3=\left(\sqrt{2}-1\right)\sqrt{x}$	M1
	$\sqrt{x}=\dfrac{3}{\left(\sqrt{2}-1\right)}\times\dfrac{\left(\sqrt{2}+1\right)}{\left(\sqrt{2}+1\right)}=3\left(\sqrt{2}+1\right)$ *	A1*
		(3)
(c)	$\sqrt{x}=3\left(\sqrt{2}+1\right)\Rightarrow x=9\left(\sqrt{2}+1\right)^2=\ldots$	M1
	$\Rightarrow x=9\left(3+2\sqrt{2}\right)$, $y=3\sqrt{2}+6$	A1, B1
		(3)
		(9 marks)

(a) **Check both diagrams and score the best single diagram unless the candidate clearly indicates which one they want marked by e.g. crossing out the other(s).** One

M1 correct curve drawn and labelled correctly:
For $\mathrm{f}(2x)$ the curve should start at O and be above and remain above $\mathrm{f}(x)$ and not head back towards it significantly i.e. at least maintain the same gap.
For $\mathrm{f}(x)+3$ the curve should start on the positive y-axis and be approximately the same shape as $\mathrm{f}(x)$

A1 One correct curve drawn as above and **labelled** and with **correct point** for that curve. The point does **not** have to be in the correct relative position – just look for values.

A1 Completely correct sketches with both points correct and at least one correctly labelled – you can assume the other is the other. Allow $\mathrm{f}(2x)$ to cross $\mathrm{f}(x)+3$ as long as it is beyond (9, 6) but with no other intersections for $x > 0$

The coordinates for the transformed *P* must be indicated on the sketch or if they are away from the sketch it must be clear which curve they relate to.

(b)

B1 Correct equation $\sqrt{x}+3=\sqrt{2x}$

M1 Writes $\sqrt{2x}$ as $\sqrt{2}\sqrt{x}$ and proceeds to collect terms in $\sqrt{x}$
Note that this may be achieved via e.g.

$$\sqrt{x}+3=\sqrt{2x}\Rightarrow\sqrt{x}+3=\sqrt{2}\sqrt{x}\Rightarrow1+\frac{3}{\sqrt{x}}=\sqrt{2}$$

Or e.g. $$\sqrt{x}+3=\sqrt{2x}\Rightarrow\sqrt{x}+3=\sqrt{2}\sqrt{x}\Rightarrow\frac{1}{3}+\frac{1}{\sqrt{x}}=\frac{\sqrt{2}}{3}$$

A1* Proceeds to the given answer showing at least the steps

$$\sqrt{x}=\frac{3}{\left(\sqrt{2}-1\right)}\times\frac{\left(\sqrt{2}+1\right)}{\left(\sqrt{2}+1\right)}=3\left(\sqrt{2}+1\right)$$

or e.g. $$3=\left(\sqrt{2}-1\right)\sqrt{x}\Rightarrow3\left(\sqrt{2}+1\right)=\left(\sqrt{2}+1\right)\left(\sqrt{2}-1\right)\sqrt{x}=\sqrt{x}$$

Attempts using e.g. $\sqrt{x}+3=\dfrac{\sqrt{x}}{2}$ score no marks in part (b)

Alternative:

B1 Correct equation $\sqrt{x}+3=\sqrt{2x}$

M1 $$\sqrt{x}+3=\sqrt{2x}\Rightarrow x+6\sqrt{x}+9=2x\Rightarrow x-6\sqrt{x}-9=0$$

$$x-6\sqrt{x}-9=0\Rightarrow\sqrt{x}=\frac{6\pm\sqrt{36+36}}{2}$$

Squares both sides and collects terms to obtain a 3TQ in $\sqrt{x}$ and attempts to solve for $\sqrt{x}$

e.g. using quadratic formula

A1* $$\frac{6\pm\sqrt{36+36}}{2}=3\pm3\sqrt{2}=3\left(\sqrt{2}\pm1\right)\Rightarrow\sqrt{x}=3\left(\sqrt{2}+1\right)$$

Simplifies and reaches the printed answer. If they give both answers score A0 but there is no requirement to explain why the other answer is rejected.

(c)

M1 Attempts to square the given expression to find x. Condone a slip on the 3 (it may remain 3) but must result in an expression of the form $\alpha+\beta\sqrt{2}$. (Working need not be shown as long as this condition is met)

A1 $x=9\left(3+2\sqrt{2}\right)$ oe such as $x=27+18\sqrt{2}$

B1 $y=3\sqrt{2}+6$

Note that $y=\sqrt{18\sqrt{2}+27}+3$ is correct but is not simplified so scores B0

Note that working for (c) must be seen in (c) i.e. do not allow working for (c) to be credited in parts (a) and (b) unless the answers are copied into (c)

4	(a)		Maximum speed of the car or model will show consumption eventually becoming negative or model may not apply for above 80 mph	B1 [1]	or, eg, doesn't drive faster than 80, or speed limit Condone eg "Maximum number of miles car can drive"
4	(b)		$\frac{d}{dv}(\frac{12}{5}v - \frac{3}{125}v^2) = 0 \quad (\Rightarrow \frac{12}{5} - \frac{6v}{125} = 0)$	M1	Attempt differentiate C & equate to 0
			$v = 50$	A1	
			$\frac{d^2}{dv^2}(\frac{12}{5}v - \frac{3}{125}v^2) = -\frac{6}{125}$ when $v = 50$ or any correct method showing that SP is a maximum	M1	Must be correct
			Maximum speed is 50 mph	A1	Units essential. Dep only on 1st M1

4	(b) ctd		**Alternative method 1**		
			$v = -\frac{b}{2a}$ $\left(= -\frac{\frac{12}{5}}{\frac{2\times(-3)}{125}}\right)$ Attempt complete square	M1	
			$v = 50$	A1	
			Coefficient of v^2 negative, hence stationary point is a maximum	M1	
			Maximum speed is 50 mph	A1	Units essential
			Alternative method 2		
			$\frac{12}{5}v - \frac{3}{125}v^2 = 0$ ($v = 0$ or 100) &	M1	Working must be seen
			Correct sketch graph & $v = 50$	B1	NB. This mark can be gained without working to justify the graph.
			$v = 50$ seen on graph as giving maximum	M1	
			Maximum speed is 50 mph	A1 **[4]**	Units essential
4	(c)		$v = 0$ does not give $C = 0$ oe	B1 **[1]**	They will not consume fuel at 0 mph oe
4	(d)		eg $k(\frac{12}{5}v - \frac{3}{125}v^2)$	B1	or "Increase both constants by the same factor" B1B1
			with any $k > 1$	B1	or with numerical value of k (> 1) B1B1 SC: "Increase both constants" B1B0
			Alternative method		
			eg $(1 + k)(\frac{12}{5}v - \frac{3}{125}v^2)$	B1	
			where $k > 0$	B1	
				[2]	

Question			Answer	Marks	AO	Guidance	
5	(a)		$x^2+y^2-6x+9y+19=0$	M1	1.1	$(x\pm3)^2\pm\ldots+\left(y\pm\frac{9}{2}\right)^2\pm\ldots+19=0$	
			$(x-3)^2-9+\left(y+\frac{9}{2}\right)^2-\frac{81}{4}+19=0$				
			$C\left(3,-\frac{9}{2}\right)$	A1	1.1	cao	
			Radius is $\frac{\sqrt{41}}{2}$	A1	1.1	cao (oe)	
				[3]			
5	(b)		$y=\frac{11}{55/4}x-11$ $\left(\Rightarrow y=\frac{4}{5}x-11\right)$	B1	2.1	Equation of line *AB* (any equivalent form) – allow unsimplified	
			$x^2+\left(\frac{4}{5}x-11\right)^2-6x+9\left(\frac{4}{5}x-11\right)+19=0$	M1*	3.1a	Substitute equation of line into equation of circle	Or $m_{CD}=-\frac{5}{4}$ (Use of $m_1m_2=-1$ with their gradient of *AB*)
			$\frac{41}{25}x^2-\frac{82}{5}x+41=0$	M1dep*	1.1	Simplify to three-term quadratic in x (or y)	Equation of *CD* is $y+\frac{9}{2}=-\frac{5}{4}(x-3)$ (using their *C* from **(a)**)
			x-coordinate of *D* is 5 (or y-coordinate of *D* is -7)	A1	1.1	**BC**	
			Area of *OBD* $=\frac{1}{2}(11)('5')$	M1	3.2a	$\frac{1}{2}(11)(x\text{-coordinate of } D)$ or other complete method	Dependent on both previous M marks
			$=27.5$	A1	1.1		
				[6]			

Question	Scheme	Marks
6(a)	(i) There is a common difference (no common ratio) and so an arithmetic series should be used.	B1
	(ii) $(u_n) = 5 + (n-1)"d"$ or $(u_n) = 5 + n"d"$	M1
	$(u_n) = 5 + 0.25(n-1)$ oe	A1
		(3)
(b)	Need $S_n = \frac{n}{2}\left(2\times 5 + (n-1)\times 0.25\right) \geqslant 350$	M1A1
	$\Rightarrow 0.25n^2 + 9.75n \geqslant 700 \Rightarrow (n+19.5)^2 - 19.5^2 \geqslant 2800 \Rightarrow n = ...$	M1
	So 37 weeks.	A1
		(4)
		(7 marks)

Notes:

(a)(i)

B1: Identifies that there is a common difference (e.g. goes up in equal amounts) between first and second, and second and third terms, or that the ratio between first and second, and second and third is not the same, and states arithmetic series/sequence to be used.

M1: Attempts general term for A.S. with n or $n-1$ used and their d (which need not be correct).

A1: Correct expression for general term, accept equivalents, eg $4.75 + 0.25n$. May use another label than u_n or no label at all.

(b)

M1: Uses the sum formula for A.S. with their a and d and equates to, or sets inequality with, 350. Accept with any inequality symbol or equality between.

A1: Correct equation/inequality. Accept with any inequality symbol or equality between.

M1: Forms and solves a 3 term quadratic (need not be gathered to one side). Any valid method (including calculator – you may need to check).

A1: 37 weeks selected as answer.

For attempts via listing, send to review.

For use of geometric series:

Case 1: If a student states arithmetic series but uses geometric series formulae, then only the B mark can be scored in (a), but in (b) allow M0A0M1A0 if a correct method is used to solve their equation $\frac{5("r"^n - 1)}{"r"-1} \geqslant 350$ (or with "=" or "<" etc) to find n (so $n = \log_{"r"}(70("r"-1)+1)$ oe)

Case 2: If a student states a geometric series and gives a common ratio, then allow B0M1A0 in (a) for $(u_n) = 5"r"^k$ with $k = n$ or $n-1$ and in (b) allow M1A0 for $\frac{5("r"^n - 1)}{"r"-1} \geqslant 350$ and then M1A0 for a correct method to solve this equation.

Question Number	Scheme		Marks
7. (a)	$\text{Area}(FEA) = \frac{1}{2}x^2\left(\frac{2\pi}{3}\right); = \frac{\pi x^2}{3}$	$\frac{1}{2}x^2 \times \left(\frac{2\pi}{3}\right)$ or $\frac{120}{360} \times \pi x^2$ simplified or un-simplified	M1
		$\frac{\pi x^2}{3}$	A1
			[2]
	Parts (b) and (c) may be marked together		
(b)	$\{A =\}\ \frac{1}{2}x^2 \sin 60^\circ + \frac{1}{3}\pi x^2 + 2xy$	Attempt to sum 3 areas (at least one correct)	M1
		Correct expression for at least two terms of A	A1
	$1000 = \frac{\sqrt{3}x^2}{4} + \frac{\pi x^2}{3} + 2xy \Rightarrow y = \frac{500}{x} - \frac{\sqrt{3}x}{8} - \frac{\pi x}{6}$ $\Rightarrow \underline{y = \frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)}$ *	Correct proof.	A1 *
			[3]
(c)	$\{P =\}\ x + x\theta + y + 2x + y \ \left\{= 3x + \frac{2\pi x}{3} + 2y\right\}$	Correct expression in x and y for their θ measured in rads	B1ft
	$...2y = \quad + 2\left(\frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)\right)$	Substitutes expression from (b) into y term.	M1
	$P = 3x + \frac{2\pi x}{3} + \frac{1000}{x} - \frac{\pi x}{3} - \frac{\sqrt{3}}{4}x \Rightarrow P = \frac{1000}{x} + 3x + \frac{\pi x}{3} - \frac{\sqrt{3}}{4}x$		
	$\Rightarrow \underline{P = \frac{1000}{x} + \frac{x}{12}\left(4\pi + 36 - 3\sqrt{3}\right)}$ *	Correct proof.	A1 *
			[3]
	Parts (d) and (e) should be marked together		
(d)	$\frac{\mathrm{d}P}{\mathrm{d}x} = -1000x^{-2} + \frac{4\pi + 36 - 3\sqrt{3}}{12}; = 0$	$\frac{1000}{x} \to \frac{\pm\lambda}{x^2}$	M1
		Correct differentiation (need not be simplified).	A1;
		Their $P' = 0$	M1
	$\Rightarrow x = \sqrt{\frac{1000(12)}{4\pi + 36 - 3\sqrt{3}}}$ $(= 16.63392808...)$	$\sqrt{\frac{1000(12)}{4\pi + 36 - 3\sqrt{3}}}$ or awrt 17 (may be implied)	A1
	$\left\{P = \frac{1000}{(16.63...)} + \frac{(16.63...)}{12}\left(4\pi + 36 - 3\sqrt{3}\right)\right\} \Rightarrow P = 120.236..\ \text{(m)}$	awrt 120	A1
			[5]
(e)	$\frac{\mathrm{d}^2P}{\mathrm{d}x^2} = \frac{2000}{x^3} > 0 \Rightarrow \text{Minimum}$	Finds P'' **and** considers sign.	M1
		$\frac{2000}{x^3}$ (need not be simplified) and > 0 and conclusion. Only follow through on a correct P'' and x in range $10 < x < 25$.	A1ft
			[2]
			15

	Question 7 Notes	
(a)	M1	Attempts to use Area$(FEA) = \frac{1}{2}x^2 \times \frac{2\pi}{3}$ (using radian angle) or $\frac{120}{360} \times \pi x^2$ (using angle in degrees)
	A1	$\frac{\pi x^2}{3}$ cao (Must be simplified and be their answer in part (a)) Answer only implies M1A1. N.B. Area$(FEA) = \frac{1}{2}x^2 \times 120$ is awarded M0A0
(b)	M1	An attempt to sum 3 " areas" consisting of rectangle, triangle and sector (allow slips even in dimensions) but **one area** should be correct
	1st A1	Correct expression for **two** of the **three** areas listed above. Accept any correct equivalents e.g. two correct from $\frac{1}{2}x^2 \sin\left(\frac{\pi}{3}\right)$ or $\frac{1}{4}x^2\sqrt{3}$, $\frac{1}{2} \times \frac{2}{3}\pi x^2$, $2xy$
	2nd A1*	This is a given answer which should be stated and should be achieved without error so all three areas must have been correct and their sum put equal to 1000 and an intermediate step of rearrangement should be present.
(c)	B1ft	Correct expression for P from arc length, length AB and three sides of rectangle in terms of both x and y with $2y$ (or $y + y$), $3x$ (or $x + 2x$) (or $x + x + x$), and $x\theta$ clearly listed . Allow addition after substitution of y. NB $\theta = \frac{2\pi}{3}$ but allow use of their consistent θ in radians (usually $\theta = \frac{\pi}{3}$) from parts (a) and (b) for this mark. $120x$ or $60x$ do not get this mark.
	M1	Substitutes $y = \frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)$ or their unsimplified attempt at y from earlier (allow slips e.g. sign slips) into $2y$ term.
	A1*	This is a given answer which should be stated and should be achieved without error
(d)	1st M1	Need to see at least $\frac{1000}{x} \to \frac{\pm\lambda}{x^2}$
	1st A1	Correct differentiation of both terms (need not be simplified) Not follow through. Allow any correct equivalent. e.g. $\frac{dP}{dx} = -1000x^{-2} + \frac{\pi}{3} + 3 - \frac{\sqrt{3}}{4}$ Also allow $\frac{dP}{dx} = -1000x^{-2} + awrt\ 3.61$ Check carefully as there are many correct equivalents and some have two terms in $x\pi$ to differentiate obtaining for example $\frac{2\pi}{3} - \frac{8\pi}{24}$ instead of $\frac{\pi}{3}$
	2nd M1	Setting their $\frac{dP}{dx} = 0$. Do not need to find x, but if inequalities are used this mark cannot be gained until candidate states or uses a value of x without inequalities. May not be explicit but may be implied by correct working and value or expression for x . May result in $x^2 < 0$ so M1A0
	2nd A1	There is no requirement to write down a value for x, so this mark may be implied by a correct value for P. It may be given for a correct expression or value for x of 16.6, 16.7 or 17
	3rd A1	Allow answers wrt 120 but not 121
(e)	M1	Finds P'' **and** considers sign. Follow through **correct** differentiation of their P' (not just reduction of power)
	A1ft	Need $\frac{2000}{x^3}$ and > 0 (or positive value) and conclusion. Only follow through on a correct P'' and a value for x in the range $10 < x < 25$ (need not see x substituted but an x should have been found) If P is substituted then this is awarded M1 A0

	Special case	(d) Some candidates multiply P by 12 to "simplify" If they write $\frac{dP}{dx} = -12000x^{-2} + 4\pi + 36 - 3\sqrt{3}$; $= 0$ then solve they will get the correct x and P They should be awarded M1A0M1A1A1 in part (d). If they then do part (e) writing $\frac{d^2P}{dx^2} = \frac{24000}{x^3} > 0 \Rightarrow$ Minimum They should be awarded M1A0 (so lose 2 marks in all) If they wrote $\frac{d(12P)}{dx} = -12000x^{-2} + 4\pi + 36 - 3\sqrt{3}$; $= 0$ etc they could get full marks.

Question Number	Scheme	Marks
8 (a)	$2\sin(\theta-30°)=5\cos\theta \Rightarrow 2\sin\theta\cos30°-2\cos\theta\sin30°=5\cos\theta$	M1
	divide by $\cos\theta$ $\quad\Rightarrow 2\tan\theta\cos30°-2\sin30°=5$	dM1
	$\Rightarrow 2\tan\theta\times\frac{\sqrt{3}}{2}-2\times\frac{1}{2}=5$	A1
	$\Rightarrow \sqrt{3}\tan\theta=6 \Rightarrow \tan\theta=2\sqrt{3}$ *	A1*
		(4)
(b)	Attempts $\arctan 2\sqrt{3}$ ..and then subtracts $20°$	M1
	$\Rightarrow x=$ awrt $53.9°, 233.9°$	A1, A1
		(3)
		(7 marks)

(a)

M1 Attempts to use $\sin(\theta-30°)=\sin\theta\cos(\pm30°)\pm\cos\theta\sin(\pm30°)$ within the given equation

Condone the omission of a 2 on the second term and a slip on the 5 of $5\cos\theta$

dM1 Divides by $\cos\theta$ to set up an equation in just $\tan\theta$.

They may collect terms in $\sin\theta$ and $\cos\theta$ before dividing by $\cos\theta$ to set up an equation in just $\tan\theta$

An equation with $\cos30°$ and $\sin30°$ still not processed is acceptable.

A1 Fully correct equation in $\tan\theta$ with the $\cos30°$ and $\sin30°$ processed

($\sqrt{3}\sin\theta=6\cos\theta \Rightarrow \tan\theta=2\sqrt{3}$ is acceptable for both A marks)

(Note If they proceed directly to the final answer from

$\tan\theta=\frac{5+2\sin30°}{2\cos30°} \Rightarrow \tan\theta=2\sqrt{3}$ then maximum M1dM1A0A0 unless

$\tan\theta=\frac{5+1}{\sqrt{3}}$ or equivalent is seen before the final given answer.

A1* Correctly proceeds to given answer.

(b) Answers with no working scores 0 marks

M1 Attempts to find a value for x.

Allow $\arctan 2\sqrt{3}$ ...followed by adding or subtracting $20°$. Which may be implied by

$\tan(x+20)=2\sqrt{3} \Rightarrow x=\arctan(2\sqrt{3})\pm20=...$

Alternatively, attempts to use $\sin\left(x-10°\right)=\sin x\cos 10°\pm\cos x\sin 10°$ within the given equation, divides by $\cos x$ to set up an equation in just $\tan x$ and proceeds to find an angle for x

$$\tan x=\frac{5\cos 20+2\sin 10}{2\cos 10+5\sin 20}\Rightarrow x=...$$

A1 One value provided M1 has been scored. Allow either awrt $54°$ or $234°$ (or in radians awrt 0.94 or 4.08)

A1 $x=\text{awrt } 53.9°, 233.9°$ and no others inside the range provided M1 has been scored. Ignore any angles outside the range. Must be in degrees

Question Number	Scheme	Marks
9(a)	$(1+2\cos 2x)^2 = 1+4\cos 2x+4\cos^2 2x$	
	Uses $\cos 4x = 2\cos^2 2x - 1 \Rightarrow (1+2\cos 2x)^2 = 1+4\cos 2x+2\cos 4x+2$	M1
	$= 3+4\cos 2x+2\cos 4x$	A1
		(2)
(b)	$a = \frac{2\pi}{3}$	B1
	$\int 3+4\cos 2x+2\cos 4x \, dx = 3x+2\sin 2x+\frac{1}{2}\sin 4x$	M1 A1 ft
	Area = $\left[3x+2\sin 2x+\frac{1}{2}\sin 4x\right]_0^{\frac{2\pi}{3}} = 2\pi - \frac{3}{4}\sqrt{3}$	dM1 A1
		(5)
		(7 marks)

(a)

M1 Attempts to multiply out $(1+2\cos 2x)^2 = 1+...\cos 2x+...\cos^2 2x$ and use $\cos 4x = 2\cos^2 2x - 1$ to obtain $(1+2\cos 2x)^2$ in the form $p+q\cos 2x+r\cos 4x$.

Condone slips in the rearrangement of $\cos 4x = 2\cos^2 2x - 1$ but it must be clear that the identity was correct originally otherwise M0.

Beware of candidates who write $(1+2\cos 2x)^2 = 1+4\cos 2x+4\cos^2 4x$ which is M0A0

A1 $3+4\cos 2x+2\cos 4x$

(b)

B1 Deduces that $a = \frac{2\pi}{3}$ (allow 120° for this mark). If more than one angle is found, then look for which one is substituted into their integrated expression.

M1 Integrates $q\cos 2x + r\cos 4x \to \pm...\sin 2x \pm ...\sin 4x$

A1ft Integrates $p+q\cos 2x+r\cos 4x \to px+\frac{q}{2}\sin 2x+\frac{r}{4}\sin 4x$ unsimplified where *p*, *q* and *r* $\neq 0$

dM1 Substitutes 0 and $a = \frac{2\pi}{3}$ (or awrt 2.09) (or equivalent) into a valid function (M1 must have been scored) and subtracts either way around. Note q, $r \neq 0$ but they do not need a px term.

Also allow $a = \frac{\pi}{3}$ (or awrt 1.05) or $a = \frac{4\pi}{3}$ (or awrt 4.19) a must be in radians to evaluate correctly.

This mark cannot be scored without a value for a. You do not have to explicitly see 0 substituted in and their answer may imply a correct substitution into their integrated expression.

A1 $2\pi - \frac{3}{4}\sqrt{3}$ or simplified equivalent

Question Number	Scheme	Marks
10(a)	$N=\dfrac{600\mathrm{e}^{0.3t}}{2+\mathrm{e}^{0.3t}} \Rightarrow \dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{\left(2+\mathrm{e}^{0.3t}\right)\times 180\mathrm{e}^{0.3t}-180\mathrm{e}^{0.3t}\times \mathrm{e}^{0.3t}}{\left(2+\mathrm{e}^{0.3t}\right)^2}$	M1 A1
	$\Rightarrow \dfrac{\mathrm{d}N}{\mathrm{d}t}=\dfrac{360\mathrm{e}^{0.3t}}{\left(2+\mathrm{e}^{0.3t}\right)^2}$	A1 **(3)**
(b)	$8=\dfrac{360\mathrm{e}^{0.3t}}{\left(2+\mathrm{e}^{0.3t}\right)^2} \Rightarrow 8\left(\mathrm{e}^{0.3t}\right)^2-328\left(\mathrm{e}^{0.3t}\right)+32=0$	M1
	$\mathrm{e}^{0.3t}=\dfrac{41+\sqrt{1665}}{2}$	dM1
	$(t=)\dfrac{\ln\left(\dfrac{41+\sqrt{1665}}{2}\right)}{0.3}$	ddM1
	$(T=)$ awrt 12.4	A1
		(4)
		(7 marks)

(a)

M1: Uses the quotient rule to obtain an expression of the form $\dfrac{\left(2+e^{0.3t}\right)\times ae^{0.3t}-be^{0.3t}\times e^{0.3t}}{\left(2+e^{0.3t}\right)^2}$

In general condone missing brackets for the M mark.

A correct rule may be implied by their *u*, *v*, *u′*, *v′* followed by applying $\dfrac{vu'-uv'}{v^2}$ **etc.**

E.g. if they quote $u = 600e^{0.3t}$ and $v = 2 + e^{0.3t}$ and do not make the differentiation easier, they can be awarded this mark for applying the correct rule. Also allow this mark if they quote the correct quotient rule but only have *v* rather than v^2 in the denominator.

A1: Correct derivative in any form e.g. $\dfrac{dN}{dt}=\dfrac{\left(2+e^{0.3t}\right)\times 180e^{0.3t}-180e^{0.3t}\times e^{0.3t}}{\left(2+e^{0.3t}\right)^2}$

A1: Correctly obtains $\dfrac{dN}{dt}=\dfrac{360e^{0.3t}}{\left(2+e^{0.3t}\right)^2}$

Withhold this mark if you see $e^{0.3t}\times e^{0.3t}$ written as $e^{0.3t^2}$ or $e^{0.09t}$

....

Alt (a)

M1 A1: $N=\dfrac{600e^{0.3t}}{2+e^{0.3t}}\Rightarrow N=600-\dfrac{1200}{2+e^{0.3t}}\Rightarrow\dfrac{dN}{dt}=\dfrac{1200\times 0.3e^{0.3t}}{\left(2+e^{0.3t}\right)^2}$

Score M1 for splitting $\dfrac{600e^{0.3t}}{2+e^{0.3t}}$ into $A\pm\dfrac{B}{2+e^{0.3t}}$ leading to $\dfrac{dN}{dt}=\dfrac{ke^{0.3t}}{\left(2+e^{0.3t}\right)^2}$

and A1 for correct derivative in any form.

A1: $\dfrac{dN}{dt}=\dfrac{360e^{0.3t}}{\left(2+e^{0.3t}\right)^2}$ (following fully correct work)

....

May also see product rule in (a):

$$N=\frac{600e^{0.3t}}{2+e^{0.3t}}=600e^{0.3t}\left(2+e^{0.3t}\right)^{-1}\Rightarrow\frac{dN}{dt}=180e^{0.3t}\left(2+e^{0.3t}\right)^{-1}-180e^{0.3t}\times e^{0.3t}\left(2+e^{0.3t}\right)^{-2}$$
$$=\frac{180e^{0.3t}}{2+e^{0.3t}}-\frac{180e^{0.6t}}{\left(2+e^{0.3t}\right)^2}=\frac{360e^{0.3t}+180e^{0.6t}-180e^{0.6t}}{\left(2+e^{0.3t}\right)^2}=\frac{360e^{0.3t}}{\left(2+e^{0.3t}\right)^2}$$

Score M1 for an expression of the form $ae^{0.3t}\left(2+e^{0.3t}\right)^{-1}-be^{0.3t}\times e^{0.3t}\left(2+e^{0.3t}\right)^{-2}$ and A1 A1 as above.

A correct rule may be implied by their *u*, *v*, *u′*, *v′* followed by applying $vu'+uv'$ **etc.**

(b)

M1: Sets $\dfrac{dN}{dt}=8$ and proceeds to a quadratic in $e^{0.3t}$

dM1: Correct attempt to solve the quadratic in $e^{0.3t}$. Condone both roots to be found

ddM1: Correct attempt to find the value of t for $e^{0.3t}=k$ where $k>0$ using correct log work

A1: Achieves $(T=)$ awrt 12.4

Question	Scheme	Marks	AOs
11(a)	$\mathrm{f}'(x)=2x+\dfrac{4x-4}{2x^2-4x+5}$	M1 A1	1.1b 1.1b
	$2x+\dfrac{4x-4}{2x^2-4x+5}=0\Rightarrow 2x\left(2x^2-4x+5\right)+4x-4=0$	dM1	1.1b
	$2x^3-4x^2+7x-2=0$ *	A1*	2.1
		(4)	
(b)	(i) $x_2=\dfrac{1}{7}\left(2+4(0.3)^2-2(0.3)^3\right)$	M1	1.1b
	$x_2=0.3294$	A1	1.1b
	(ii) $x_4=0.3398$	A1	1.1b
		(3)	
(c)	$\mathrm{h}(x)=2x^3-4x^2+7x-2$ $\mathrm{h}(0.3415)=0.00366...$ $\mathrm{h}(0.3405)=-0.00130...$	M1	3.1a
	States: • there is a change of sign • $\mathrm{f}'(x)$ is continuous • $\alpha=0.341$ to 3dp	A1	2.4
		(2)	
			(9 marks)

Notes

(a)

M1: Attempt to differentiate $\ln\left(2x^2-4x+5\right)$ to obtain $\frac{g(x)}{2x^2-4x+5}$ where g(x) could be 1

A1: For $f'(x)=2x+\frac{4x-4}{2x^2-4x+5}$

dM1: Sets their f′(x) $=ax+\frac{g(x)}{2x^2-4x+5}=0$ and uses "**correct**" algebra, condoning slips, to obtain a cubic equation. E.g Look for $ax\left(2x^2-4x+5\right)\pm g(x)=0$ o.e. , condoning slips, followed by some attempt to simplify

A1*: Achieves $2x^3-4x^2+7x-2=0$ with no errors. (The dM1 mark must have been awarded)

(b)(i)

M1: Attempts to use the iterative formula with $x_1=0.3$. If no method is shown award for $x_2=\text{awrt } 0.33$

A1: $x_2=\text{awrt } 0.3294$ Note that $\frac{1153}{3500}$ is correct

Condone an incorrect suffix if it is clear that a correct value has been found

(b)(ii)

A1: $x_4=\text{awrt } 0.3398$ Condone an incorrect suffix if it is clear that a correct value has been found

(c)

M1: Attempts to substitute x = 0.3415 and x = 0.3405 into a suitable function and gets one value correct (rounded or truncated to 1 sf). It is allowable to use a tighter interval that contains the root 0.340762654

Examples of suitable functions are $2x^3-4x^2+7x-2,\ x-\frac{1}{7}\left(4x^2-2x^3+2\right)$ and f '(x) as this has been found in part (a) with f '(0.3405)= - 0.00067.., f '(0.3415)= (+) 0.00189

There must be sufficient evidence for the function, which would be for example, a statement such as $h(x)=2x^3-4x^2+7x-2$ or sight of embedded values that imply the function, not just a value or values even if both are correct. Condone $h(x)$ being mislabelled as f

$h(0.3415)=2\times0.3415^3-4\times0.3415^2+7\times0.3415-2$

A1: Requires

- both calculations correct (rounded or truncated to 1sf)
- a statement that there is a change in sign and that the function is continuous
- a minimal conclusion e.g. ✓, proven, α = 0.341, root

Question	Scheme	Marks	AOs
12(a)	$\int_4^{12} (1+2x)^{\frac{1}{2}}\,\mathrm{d}x$	B1	1.2
		(1)	
(b)	$\int (1+2x)^{\frac{1}{2}}\,\mathrm{d}x = k(1+2x)^{\frac{3}{2}}(+c)$	M1	1.1b
	$\int (1+2x)^{3}\,\mathrm{d}x = \frac{1}{3}(1+2x)^{\frac{3}{2}}(+c)$	A1	1.1b
	$\int_4^{12} (1+2x)^{\frac{1}{2}}\,\mathrm{d}x = \left[\frac{1}{3}(1+2x)^{\frac{3}{2}}\right]_4^{12} = \frac{1}{3}(125-27) = \frac{98}{3}$	A1	1.1b
		(3)	
			(4 marks)

Notes

(a)

B1: States $\int_4^{12} (1+2x)^{\frac{1}{2}}\,\mathrm{d}x$

(b)

M1: Integrates $(1+2x)^{\frac{1}{2}}$ to give $k(1+2x)^{\frac{3}{2}}(+c)$

A1: $\int (1+2x)^{\frac{1}{2}}\,\mathrm{d}x = \frac{1}{3}(1+2x)^{\frac{3}{2}}(+c)$

A1: Fully correct proof with evidence of correct use of both limits

Question Number	Scheme	Marks
13	$\int x^2 \ln x \, dx = \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^3 \times \frac{1}{x} \, dx$	M1
	$= \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 \, (+c)$	dM1 A1
		(3)
		(3 marks)

Note that no marks are available for integrating incorrect functions

M1: Attempts to integrate by parts the correct way around. Look for

$$\int x^2 \ln x \, dx = ...x^3 \ln x - \int ...x^3 \times \frac{1}{x} \, dx$$ oe

dM1: And then integrates again to a form $px^3 \ln x - qx^3$ where p and q are positive constants.

A1: $\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 + c$ Condone a missing + c.

Candidates who go on to write $\ln x \times \frac{x^3}{3} - \frac{1}{9}x^3 = \ln \frac{x^4}{3} - \frac{1}{9}x^3$ o.e. withhold this final A1

Question	Scheme	Marks	AOs
14(a)	$\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}=-3\mathbf{i}-4\mathbf{j}-5\mathbf{k}+\mathbf{i}+\mathbf{j}+4\mathbf{k}=...$	M1	1.1b
	$=-2\mathbf{i}-3\mathbf{j}-\mathbf{k}$	A1	1.1b
		(2)	
(b)	At least 2 of $\left(AC^2\right)="2^2+3^2+1^2",\left(AB^2\right)=3^2+4^2+5^2,\left(BC^2\right)=1^2+1^2+4^2$	M1	1.1b
	$2^2+3^2+1^2=3^2+4^2+5^2+1^2+1^2+4^2-2\sqrt{3^2+4^2+5^2}\sqrt{1^2+1^2+4^2}\cos ABC$	M1	3.1a
	$14=50+18-2\sqrt{50}\sqrt{18}\cos ABC$ $\Rightarrow \cos ABC=\dfrac{50+18-14}{2\sqrt{50}\sqrt{18}}=\dfrac{9}{10}*$	A1*	2.1
		(3)	
	(b) Alternative		
	$AB^2=3^2+4^2+5^2,\ BC^2=1^2+1^2+4^2$	M1	1.1b
	$\overrightarrow{BA}.\overrightarrow{BC}=(3\mathbf{i}+4\mathbf{j}+5\mathbf{k})\bullet(\mathbf{i}+\mathbf{j}+4\mathbf{k})=27=\sqrt{3^2+4^2+5^2}\sqrt{1^2+1^2+4^2}\cos ABC$	M1	3.1a
	$27=\sqrt{50}\sqrt{18}\cos ABC\Rightarrow\cos ABC=\dfrac{27}{\sqrt{50}\sqrt{18}}=\dfrac{9}{10}*$	A1*	2.1
			(5 marks)
	Notes		

(a)

M1: Attempts $\overrightarrow{AC}=\overrightarrow{AB}+\overrightarrow{BC}$

There must be attempt to add not subtract.

If no method shown it may be implied by **two** correct components

A1: Correct vector. Allow $-2\mathbf{i}-3\mathbf{j}-\mathbf{k}$ and $\begin{pmatrix}-2\\-3\\-1\end{pmatrix}$ but not $\begin{pmatrix}-2\mathbf{i}\\-3\mathbf{j}\\-1\mathbf{k}\end{pmatrix}$

(b)

M1: Attempts to "square and add" for at least 2 of the 3 sides. Follow through on their $\overrightarrow{AC}$

Look for an attempt at either $a^2+b^2+c^2$ or $\sqrt{a^2+b^2+c^2}$

M1: A correct attempt to apply a correct cosine rule to the given problem; Condone **slips** on the lengths of the sides but the sides must be in the correct position to find angle ABC

A1*: Correct completion with sufficient intermediate work to establish the printed result.

Condone different labelling, e.g. $ABC\leftrightarrow\theta$ as long as it is clear what is meant

It is OK to move from a correct cosine rule $14=50+18-2\sqrt{50}\sqrt{18}\cos ABC$

via $\cos ABC=\dfrac{54}{2\sqrt{50}\sqrt{18}}$ o.e. such as $\cos ABC=\dfrac{\left(5\sqrt{2}\right)^2+\left(3\sqrt{2}\right)^2-\left(\sqrt{14}\right)^2}{2\times5\sqrt{2}\times3\sqrt{2}}$ to $\cos ABC=\dfrac{9}{10}$

Alternative:

M1: Correct application of Pythagoras for sides AB and BC or their squares

M1: Recognises the requirement for and applies the scalar product

A1*: Correct completion with sufficient intermediate work to establish the printed result