Mark Scheme

Q1.

Question Number	Scheme	Marks
	$\log_4 \frac{a}{b} = 3$ or $\log_4 a + \log_4 b = \log_4 25$ or $\log_4 \frac{a}{\frac{25}{a}} = 3$ or $\log_4 \frac{\frac{25}{b}}{b} = 3$ (If this is preceded by wrong algebra (e.g. b = 25 –a) M1 can still be given if their b is used	M1
	$\log_4 64 = 3$ or $4^3 = 64$ (may be implied by the use of 64) or see $\log a = \frac{1}{2}(\log 25 + 3)$ become $a = 4^{\frac{1}{2}(\log 25+3)}$ or see $\log b = \frac{1}{2}(\log 25 - 3)$ become $b = 4^{\frac{1}{2}(\log 25-3)}$ (these latter two statements will be implied by correct answers)	B1
	Correct algebraic elimination of a variable to obtain expression in a or b without logs	dM1
	$a = 40$ **or** $b = \frac{5}{8}$	A1
	Substitutes to give second variable or solves again from start	dM1
	$a = 40$ **and** $b = \frac{5}{8}$ and no other answers.	A1
		[6]
		6 marks
	Notes	
	M1: Uses addition or subtraction law correctly for logs (N.B. $\log_4 a + \log_4 b = 25$ is **M0**) **B1**: See number 64 used (independent of M mark) or or see $\log a = \frac{1}{2}(\log 25 + 3)$ become $a = 4^{\frac{1}{2}(\log 25+3)}$ or see $\log b = \frac{1}{2}(\log 25 - 3)$ become $b = 4^{\frac{1}{2}(\log 25-3)}$ **dM1**: Dependent on first M mark. Eliminates a or b (with appropriate algebra) and eliminates logs **A1**: Either a or b correct **dM1**: Dependent on first M mark . Attempts to find second variable **A1**: Both a and b correct – allow $b = 0.625$ If a = -40 and b = -5/8 are **also** given as answers **lose the last A mark**. **NB** Log a + log b = 2.3219..will not yield exact answers If they round their answers to 40 and 0.625 after decimal work, do not give final A mark. NB: Some will change the base of the log and use loga – log b = 3log4	

Q2.

Question Number	Scheme	Marks
(a)	$u_2 = -2,\ u_3 = -7$ and $u_4 = -12$	M1, A1 [2]
(b)	$d = -5$ and arithmetic	B1
	Uses $a + (n-1)d$ with $a = 3$ and $n = 100$, to give -492	M1, A1 [3]
(c)	$S_{100} = \frac{n}{2}(2a + (n-1)d)\ or\ \frac{n}{2}(a+l)$	M1
	$S_{100} = \frac{100}{2}(6 + 99 \times -5)\ or\ \frac{100}{2}(3 + -492)$	dM1
	$= -24\,450$	A1 [3]
		8 marks
	Notes	

(a)

M1: Attempt to use formula correctly at least twice. ("Subtract 5") Follow through on an incorrect u_2 or u_3

A1: three correct answers

(b)

B1: Assumes AP and uses or states that $d = -5$. Hence B0 if you see for example $d = -5$, followed by $3 \times (-5)^{99}$

You may assume an AP if you see any AP formula.

M1: Correct formula used and processed correctly. Look for $3 + 99 \times "d"$ or $-2 + 98 \times "d"$ with their d.

The (n -1) must be multiplied by d.

So, students that write $S_{100} = a + (n-1)d = 3 + (100-1) - 5 = 97$ score B1 M0 A0 for incorrect processing

A1: -492 (cao)

(c)

M1: States or uses a correct sum formula for an AP with $n = 100$ with any values for a, d and l

dM1: Uses and processes a correct sum formula for an AP with $a = 3\ or -2$, $d = \pm 5$ and ft on their l

Note that students who write $S_{100} = \frac{n}{2}(2a + (n-1)d) = \frac{100}{2}(6 + (100-1) - 5) = 5000$ score M1 dM0 A0

A1: Obtains $-24\,450$

Q3.

Question Number	Scheme	Marks
	$\left(2-\frac{x}{2}\right)^6 = 2^6 + \binom{6}{1}2^5.\left(-\frac{x}{2}\right)+\binom{6}{2}2^4.\left(\frac{-x}{2}\right)^2+\ldots$	M1
	$= 64, -96x, +60x^2+\ldots$	B1, A1, A1
		[4]
	Special case $= 64, -192\left(\frac{x}{2}\right), +240\left(\frac{x}{2}\right)^2+\ldots$ This is correct but unsimplified M1B1A1A0	
		4 marks
Alternative method	$[2^6]\left(1-\frac{x}{4}\right)^6 = [2^6]\left(1+\binom{6}{1}\left(-\frac{x}{4}\right)+\binom{6}{2}\left(\frac{-x}{4}\right)^2+\ldots\right)$	M1
	$= 64, -96x, +60x^2+\ldots$	B1, A1, A1
	Notes	
	M1: The **method** mark is awarded for an attempt at Binomial to get the second **and/or** third term – need **correct** binomial coefficient combined with correct power of *x*. Ignore bracket errors or errors (or omissions) in powers of 2 or sign or bracket errors. Accept any notation for ${}^6C_1$ and ${}^6C_2$, e.g. $\binom{6}{1}$ and $\binom{6}{2}$ (unsimplified) or 6 and 15 from Pascal's triangle This mark may be given if no working is shown, but either or both of the terms including *x* is correct. **B1:** must be simplified to 64 (writing just 2^6 is **B0**). This must be the only constant term (do not isw here) **A1:** is cao and is for $-96x$. The *x* is required for this mark. Allow +(-96*x*) **A1:** is cao and is for $60x^2$ (can follow omission of negative sign in working) Any extra terms in higher powers of *x* should be ignored **Isw** if this is followed by $=16, -24x, +15x^2+\ldots$ Allow terms separated by commas and given as list <u>Alternative Method</u> M1: Does not require power of 2 to be accurate B1: If answer is left as $64\left(1+\binom{6}{1}\left(-\frac{x}{4}\right)+\binom{6}{2}\left(\frac{-x}{4}\right)^2+\ldots\right)$ Allow M1 B1 A0 A0	

Q4.

Question Number	Scheme	Marks
(a)	$R=\sqrt{4+16}=\sqrt{20}\ or\ 2\sqrt{5}$	B1
	$\tan\alpha=\frac{4}{2}$	M1
	$\Rightarrow \alpha=1.11$ (awrt)	A1
		(3)
(b)	Maximum is 12+2R or minimum is 12– 2R	M1
	maximum = 20.9 (hours) (20h 57m) and minimum = 3.06 (hours) (3 hours 3 m)	A1 A1
		(3)
(c)	$17=12+k"R"\sin\left(\frac{2\pi t}{365}\pm"\alpha"\right)$	M1
	$\sin\left(\frac{2\pi t}{365}\pm"\alpha"\right)=....$	dM1
	For proceeding to one value for t from $17=12+2"R"\sin\left(\frac{2\pi t}{365}\pm"\alpha"\right)$	M1
	$t=99$ (days) **or** 212 or 213 (days)	A1
	For finding two values for t	dM1
	$t=99$ (days) **and** 212 or 213 (days)	A1
		(6)
		(12 marks)

(a)

B1: $R=\sqrt{20}$ or $2\sqrt{5}$ no working needed. Condone $R=\pm\sqrt{20}$ oe

M1: $\tan\alpha=\pm\frac{4}{2}$ or $\tan\alpha=\pm\frac{2}{4}$ and attempts to find alpha. If R is used accept $\sin\alpha=\pm\frac{4}{"R"}$ or $\cos\alpha=\pm\frac{2}{"R"}$

A1: accept $\alpha=$ awrt 1.11 ; also accept $\sqrt{20}\sin(x-1.11)$. Answers in degrees are A0

(b)

M1: Uses Maximum is 12+2R or minimum is 12– 2R with their value of R

A1: maximum value or minimum value correct allowing exact value(s) $12\pm2\sqrt{20}$ or $12\pm4\sqrt{5}$

A1: maximum **and** minimum value **awrt** 20.9 (20h 57m) 3.06 (3 hours 3 m)

Ignore any units in this part.

Note: It is possible to do this by differentiation. To score M1 you would need to see

Differentiation to $\lambda\cos\left(\frac{2\pi t}{365}-'\alpha'\right)=0\Rightarrow\frac{2\pi t}{365}-'\alpha'=\frac{\pi}{2}$ or $\frac{3\pi}{2}\Rightarrow t=...$ and then substitute into H and find a value.

(c)

M1: For an attempt to interpret the model and writing it in terms of (a), condoning slips

Allow for $17 = 12 + k"R"\sin\left(\frac{2\pi t}{365} \pm "\alpha"\right)$, even $k = 1$ with their value for R and α (Slip on "2")

Allow $17 = 12 + k"R"\sin\left(x \pm "\alpha"\right)$ even $k = 1$ with their value for R and α (x instead of $\frac{2\pi t}{365}$)

dM1: For attempting to make $\sin(x \pm \text{their } \alpha)$ or $\sin\left(\frac{2\pi t}{365} \pm "\alpha"\right)$ the subject.

M1: For the method of finding at least one value for t, $0 < t < 365$, from a "**correct**" starting point with $2 \times$ their R.

$17 = 12 + 2"R"\sin\left(\frac{2\pi t}{365} \pm "\alpha"\right) \to \sin\left(\frac{2\pi t}{365} \pm "\alpha"\right) = C$ to $t = ...$ by undoing the operations in the correct order

A good intermediate value to check (for correct R) is $\frac{2\pi t}{365} \pm "\alpha" = 0.593....$

Condone slips on the $\frac{2\pi}{365}$ for all M marks. Example you may see $\frac{2\pi}{36}$

A1: For one correct value for t, either awrt 99 **or** awrt 212/213.

dM1: For attempting to find a second value for t.

It is dependent upon the previous M mark and it is usually for moving from

$\left(\frac{2\pi t}{365} \pm "\alpha"\right) = \pi - \beta$ (where β was the principal value) to $t = ...$

by undoing the operations in the correct order

A good intermediate value to check (for correct R) is $\frac{2\pi t}{365} \pm "\alpha" = 2.548....$

A1: awrt 99 **and** awrt 212 or 213 only $0 < t < 365$. Remember to ISW

Q5.

Question Number	Scheme	Marks
(a)	$2x^2+x+6=mx-2 \Rightarrow 2x^2+x-mx+8\ (=0)$	M1
	$b^2-4ac<0 \Rightarrow (1-m)^2-4(2)(8)<0$	M1
	$m^2-2m-63<0$*	A1*
		(3)
(b)	$m^2-2m-63=0 \Rightarrow (m-9)(m+7)=0 \Rightarrow m=...$	M1
	$(m=)\ \ -7, 9$	A1
	Attempt at inside region	M1
	$-7<m<9$	A1
		(4)
		(7 marks)

(a)

M1 Sets $2x^2+x+6=mx-2$ and rearranges the equation $(=0)$. Condone sign slips but all terms must be on one side. The two terms in x do not have to be collected together for this mark and the "=0" may be implied by further work.

M1 Attempts $b^2-4ac...0$ using values of $a=\pm 2,\ b=\pm 1\pm m,\ c=\pm 8$ or ± 4 where an inequality or equals sign is used. You may see $b^2...4ac$. It is sufficient to see the values substituted in correctly for this mark and you can condone invisible brackets. They must have achieved a quadratic in x to calculate the discriminant.

A1* Proceeds to given answer with no errors. The correct inequality for the discriminant must appear before the final line.

(b)

M1 Attempts to solve the given quadratic (which may be in terms of another variable) to find at least one of the critical values for m. Apply general marking principles for solving a quadratic.

A1 $(m=)-7, 9$ only

M1 Finds inside region for their critical values. They may draw a diagram but they must proceed to an $...<m<...$ (allow use of $\le$ for one or both inequalities for this mark and they may even be separate statements). May be in terms of x or any other variable for this mark.

A1 $-7<m<9$ **Must be in terms of m**

Accept others such as , $m>-7$ AND $m<9$ $\ \ m>-7,\ \text{m}<9$, $9>m>-7$,$\{m: -7<m<9\}$

and also accept $(-7, 9)$

Do NOT accept $m>-7$ OR $\text{m}<9$ and do NOT accept inequalities shown on a number line.

Correct answers with no working is 4/4 in (b)

Q6.

Question Number	Scheme		Marks
	Note that use of ln kx for ln x is acceptable throughout.		
(a)	$\int \frac{4x+3}{x}\,dx \to \int ...+\frac{b}{x}\,dx = ..+..\ln x$ Attempts to divide to obtain $...+\frac{b}{x}$ and uses $\int \frac{1}{x}\,dx = \ln x$ or $\int \frac{1}{x}\,dx = \ln kx$		M1
	$= 4x + 3\ln x + (c)$	There is no requirement for the + c	A1
			(2)
(a) Way 2	$\int \frac{4x+3}{x}\,dx = \int (4x+3)x^{-1}\,dx = (4x+3)\ln x - \int 4\ln x\,dx$ $\int \frac{4x+3}{x}\,dx = (4x+3)\ln x - \int ...\ln x\,dx = (4x+3)\ln x - 4x\ln x + kx$ This method requires 2 applications of parts to obtain an expression of this form		M1
	$= (4x+3)\ln x - 4x\ln x + 4x(+c)$	There is no requirement for the + c	A1
(a) Way 3	$\int \frac{4x+3}{x}\,dx = \int (4x+3)x^{-1}\,dx = (2x^2+3x)x^{-1} + \int (2x^2+3x)x^{-2}\,dx$ $= (2x+3) + \int (2+3x^{-1})\,dx = 2x+3+2x+3\ln x(+c)$ $\int \frac{4x+3}{x}\,dx = (2x^2+3x)x^{-1} + ... + ...\ln x$ This method requires the applications of parts to obtain an expression of this form		M1
	$= (2x^2+3x)x^{-1} + 2x + 3\ln x(+c)$	There is no requirement for the + c	A1

(b)	$\frac{dy}{dx} = \frac{(4x+3)y^{\frac{1}{2}}}{x} \Rightarrow \int \frac{1}{y^{\frac{1}{2}}}dy = \int \frac{(4x+3)}{x}dx$ Separates the variables correctly. Accept $\int \frac{1}{y^{\frac{1}{2}}}dy = \int \frac{(4x+3)}{x}dx$ or equivalent. With or without the integral signs and possibly without the "dx" and/or "dy" so look for $\frac{1}{y^{\frac{1}{2}}} = \frac{(4x+3)}{x}$		B1
	$2y^{\frac{1}{2}} = 4x + 3\ln x + c$	Look for $ky^{\frac{1}{2}}$ = their (a) or $ky^{\frac{1}{2}}$ = an attempt at $\int \frac{4x+3}{x}\,dx$	M1
		$2y^{\frac{1}{2}} = 4x + 3\ln x + c$ or equivalent including the $+c$	A1
	$x = 1, y = 25$ $\Rightarrow 2(25)^{\frac{1}{2}} = 4(1) + 3\ln(1) + c \Rightarrow c = ...$	Substitutes $x = 1$ and $y = 25$ into their integrated equation and proceeds to obtain a value for c.	M1
	$y = \left(2x + \frac{3}{2}\ln x + 3\right)^2$	Correct equation including "$y =$". The $2x + \frac{3}{2}\ln x + 3$ can be in any equivalent correct form.	A1
			(5)
			[7 marks]

Q7.

Question Number	Scheme	Notes	Marks
	$\frac{d(4x\sin x)}{dx} = 4x\cos x + 4\sin x$	Applies product rule to $4x\sin x$ to give $\frac{d(4x\sin x)}{dx} = \pm 4x\cos x + 4\sin x$	M1
	$\frac{d(\pi y^2)}{dy} = 2\pi y\frac{dy}{dx}$	Applies chain rule to πy^2 to give $\frac{d(\pi y^2)}{dy} = Ay\frac{dy}{dx}$	M1
	$4x\sin x = \pi y^2 + 2x \Rightarrow 4x\cos x + 4\sin x = 2\pi y\frac{dy}{dx} + 2$ Fully correct differentiation. oe Accept $4x\cos x dx + 4\sin x dx = 2\pi y dy + 2dx$		A1
	For the differentiation ignore any spurious $"\frac{dy}{dx} ="$		

	Alternative for first 3 marks using explicit differentiation: $y = \left(\frac{1}{\sqrt{\pi}}\right)(4x\sin x - 2x)^{\frac{1}{2}}$	
	$\frac{dy}{dx} = \left(\frac{1}{2\sqrt{\pi}}\right)(4x\sin x - 2x)^{-\frac{1}{2}}(4x\cos x + 4\sin x - 2)$ M1: $\frac{d(4x\sin x)}{dx} = \pm 4x\cos x + 4\sin x$ (as before) M1: $(4x\sin x - 2x)^{\frac{1}{2}} \to k(4x\sin x - 2x)^{-\frac{1}{2}}$	M1 M1
	Allow omission of π and sign errors when rearranging for the M marks	
	$\frac{dy}{dx} = \frac{1}{2\sqrt{\pi}}(4x\sin x - 2x)^{-\frac{1}{2}}(4x\cos x + 4\sin x - 2)$ oe	A1

	$x = \frac{\pi}{2}, y = 1$ $\Rightarrow 4 = 2\pi\frac{dy}{dx} + 2 \Rightarrow \frac{dy}{dx} = ...\left(\frac{1}{\pi}\right)$	Uses $x = \frac{\pi}{2}$ and $y = 1$ to obtain a value for $\frac{dy}{dx}$ (may be implied). For implicit differentiation, there must be a dy/dx and there must be x's and y's. Explicit differentiation just requires use of $x = \frac{\pi}{2}$.	M1
	$y - 1 = "-\pi"\left(x - \frac{\pi}{2}\right)$ or $y = "-\pi"x + c \Rightarrow c = 1 + \frac{\pi^2}{2}$ Uses normal gradient $-1/\frac{dy}{dx}$ and $x = \frac{\pi}{2}, y = 1$ to find equation of normal. Must use $-1/\left(\text{their } \frac{dy}{dx}\right)$ and $x = \frac{\pi}{2}$ and $y = 1$ must be correctly placed. If using $y = mx + c$ must reach as far as $c = \ldots$		M1
	$y - 1 = -\pi\left(x - \frac{\pi}{2}\right)$ oe	Allow 3sf or more decimal equivalent answers e.g. $y = -3.14x + 5.93$, $y - 1 = -3.14(x - 1.57)$ etc.	A1cso
			(6 marks)

Q8.

Qu	Scheme	Marks
(i)	$\frac{dy}{dx} = 5x^2 \times \frac{3}{3x} + \ln(3x) \times 10x$	M1 A1
		(2)
(ii)	$\frac{dy}{dx} = \frac{(\sin x + \cos x)1 - x(\cos x - \sin x)}{(\sin x + \cos x)^2}$	M1
	$\frac{dy}{dx} = \frac{(\sin x + \cos x)1 - x(\cos x - \sin x)}{(\sin^2 x + \cos^2 x) + (2\sin x \cos x)} = \frac{(\sin x + \cos x)1 - x(\cos x - \sin x)}{1 + \sin 2x}$	B1, B1
	$\frac{dy}{dx} = \frac{(1+x)\sin x + (1-x)\cos x}{1 + \sin 2x}$ *	A1 *
		(4)
		(6 marks)

(i)
M1: Applies the Product rule to $y = 5x^2 \ln 3x$

Expect $\frac{dy}{dx} = Ax + Bx\ln(3x)$ for this mark (A, B positive constant)

A1: cao- need not be simplified

(ii)
M1: Applies the Quotient rule, a form of which appears in the formula book, to $y = \frac{x}{\sin x + \cos x}$

Expect $\frac{dy}{dx} = \frac{(\sin x + \cos x)1 - x(\pm\cos x \pm \sin x)}{(\sin x + \cos x)^2}$ for M1

Condone invisible brackets for the M and an attempted incorrect 'squared' term on the denominator
Eg $\sin^2 x + \cos^2 x$

B1: Denominator should be expanded to $\sin^2 x + \cos^2 x + \ldots$ and $(\sin^2 x + \cos^2 x) \to 1$

B1: Denominator should be expanded to $\ldots + k\sin x \cos x$ and $(k\sin x \cos x) \to \frac{k}{2}\sin 2x$.

For example sight of $(\sin x + \cos x)^2 = 1 + 2\sin x \cos x = 1 + \sin 2x$ without the intermediate line on the Denominator is B0 B1

A1: cso – answer is given. This mark is withheld if there is poor notation $\cos x \leftrightarrow \cos$ $\sin^2 x \leftrightarrow \sin x^2$
If the only error is the omission of $(\sin^2 x + \cos^2 x) \to 1$ then this final A1* can be awarded.

Use of product rule or implicit differentiation needs to be applied correctly with possible sign errors differentiating functions for M1, then other marks as before. If quoted the product rule must be correct

Product rule $\frac{dy}{dx} = (\sin x + \cos x)^{-1} \times 1 \pm x \times (\sin x + \cos x)^{-2}(\pm\cos x \pm \sin x)$

Implicit differentiation $(\sin x + \cos x)y = x \Rightarrow (\sin x + \cos x)\frac{dy}{dx} + y(\pm\cos x \pm \sin x) = 1$

To score the B's under this method there must have been an attempt to write $\frac{dy}{dx}$ as a single fraction

Q9.

Question Number	Scheme	Marks
	$x = 2\sin\theta \Rightarrow \frac{dx}{d\theta} = 2\cos\theta$	B1
	$\int \frac{1}{(4-x^2)^{3/2}}dx = \int \frac{1}{(4-4\sin^2\theta)^{3/2}} 2\cos\theta(d\theta)$	M1
	$= \int \frac{1}{4}\sec^2\theta(d\theta)$ *OR* $\int \frac{1}{4}\times\frac{1}{\cos^2\theta}(d\theta)$	M1
	$= \frac{1}{4}\tan\theta$	dM1A1
	Uses limits 0 and $\frac{\pi}{3}$ in their integrated expression	
	$= \left[\frac{1}{4}\tan\theta\right]_0^{\frac{\pi}{3}} = \frac{\sqrt{3}}{4}$	M1A1
		(7 marks)

B1 States either $\frac{dx}{d\theta} = 2\cos\theta$ or $dx = 2\cos\theta\, d\theta$. Condone $x' = 2\cos\theta$

M1 Attempt to produce integral in just θ by substituting $x = 2\sin\theta$ and using $dx = \pm A\cos\theta\ (d\theta)$
You may condone a missing $d\theta$

M1 Uses $1-\sin^2\theta = \cos^2\theta$ and simplifies integral to $\int C\sec^2\theta(\,d\theta)$ or $\int \frac{C}{\cos^2\theta}(\,d\theta)$

Again you may condone a missing $d\theta$

dM1 Dependent upon previous M1 for $\int \sec^2\theta \to \tan\theta$

A1 $\frac{1}{4}\tan\theta(+c)$. No requirement for the $+c$

M1 Changes limits in x to limits in θ of 0 and $\frac{\pi}{3}$, then subtracts their integrated expression either way around. The subtraction of 0 can be implied if f (0) =0. If the candidate changes the limits to 0 and 60 (degrees) it scores M0, A0. Alternatively they could attempt to change their integrated expression in θ back to a function in x and use the original limits. Such a method would require seeing either $\cos\theta = \sqrt{1-\frac{x^2}{4}}$ or $\tan\theta = \frac{\frac{x}{2}}{\sqrt{1-\frac{x^2}{4}}}$

A1 $\frac{\sqrt{3}}{4}$.

Q10.

Question Number	Scheme	Marks
	Examples:	
(a)	$\frac{1-\cos 2x}{1+\cos 2x}=\frac{1-(1-2\sin^2 x)}{1+(2\cos^2 x-1)}=\frac{2\sin^2 x}{2\cos^2 x}=\tan^2 x$	M1dM1A1
	$\frac{1-\cos 2x}{1+\cos 2x}=\frac{1-(1-2\sin^2 x)}{1+(2\cos^2 x-1)}=\frac{\sin^2 x}{\cos^2 x}=\tan^2 x$	M1dM1A0
	$\frac{1-\cos 2x}{1+\cos 2x}=\frac{1-\cos^2 x+\sin^2 x}{1+\cos^2 x-\sin^2 x}=\frac{2\sin^2 x}{2\cos^2 x}=\tan^2 x$	M1dM1A1
	$\frac{1-\cos 2x}{1+\cos 2x}=\frac{1-\cos^2 x+\sin^2 x}{1+\cos^2 x-\sin^2 x}=\frac{\sin^2 x}{\cos^2 x}=\tan^2 x$	M1dM1A0
	$\frac{1-\cos 2x}{1+\cos 2x}=\frac{2\sin^2 x}{2\cos^2 x}=\tan^2 x$	M1dM1A1
	$\frac{1-\cos 2x}{1+\cos 2x}=\frac{\sin^2 x}{\cos^2 x}=\tan^2 x$	M0dM0A0
	$\frac{1-\cos 2x}{1+\cos 2x}=\frac{1-\cos^2 x+\sin^2 x}{1+\cos^2 x-\sin^2 x}$ $=\frac{\cos^2 x+\sin^2 x-\cos^2 x+\sin^2 x}{2\cos^2 x}=\frac{\sin^2 x}{\cos^2 x}=\tan^2 x$	M1dM1A0
		(3)

(b)	$\frac{2-2\cos 2\theta}{1+\cos 2\theta}-2=7\sec\theta$	
	$2\left(\frac{1-\cos 2\theta}{1+\cos 2\theta}\right)-2=7\sec\theta \Rightarrow 2\tan^2\theta-2=7\sec\theta$	M1
	$\Rightarrow 2(\sec^2\theta-1)-2=7\sec\theta$	M1
	$\Rightarrow 2\sec^2\theta-7\sec\theta-4=0$	A1
	$\Rightarrow (2\sec\theta+1)(\sec\theta-4)=0$	
	$\Rightarrow \sec\theta=-\frac{1}{2},4$	
	$\Rightarrow \cos\theta=-2,\frac{1}{4}\Rightarrow\theta=\ldots$	M1
	$\Rightarrow \theta=75.5°,-75.5°$	A1,A1
		(6)
		(9 marks)
(a) alt1	$\tan^2 x=\frac{\sin^2 x}{\cos^2 x}=\frac{\frac{1}{2}(1-\cos 2x)}{\frac{1}{2}(1+\cos 2x)}=\frac{(1-\cos 2x)}{(1+\cos 2x)}$	M1dM1A1
(a) alt2	$\frac{1-\cos 2x}{1+\cos 2x}=\tan^2 x\Rightarrow 1-\cos 2x=\tan^2 x(1+\cos 2x)$ $1-(1-2\sin^2 x)=\tan^2 x(1+2\cos^2 x-1)$ $2\sin^2 x=\frac{\sin^2 x}{\cos^2 x}(2\cos^2 x)$ $2\sin^2 x=2\sin^2 x$	M1dM1A1

Question Number	Scheme	Marks
	Examples:	
	Which is true*	

(b)alt1	$2\left(\frac{1-\cos 2\theta}{1+\cos 2\theta}\right)-2=7\sec\theta \Rightarrow 2\tan^2\theta-2=7\sec\theta$	M1
	$\Rightarrow 2\frac{\sin^2\theta}{\cos^2\theta}-2=\frac{7}{\cos\theta}$ $\Rightarrow 2\sin^2\theta-2\cos^2\theta=7\cos\theta$ $\Rightarrow 2(1-\cos^2\theta)-2\cos^2\theta=7\cos\theta$	M1
	$\Rightarrow 4\cos^2\theta+7\cos\theta-2=0$	A1
	$\Rightarrow (4\cos\theta-1)(\cos\theta+2)=0$	
	$\Rightarrow \cos\theta=-2,\frac{1}{4}\Rightarrow\theta=...$	M1
	$\Rightarrow \theta=75.5°,-75.5°$	A1A1
		(6)
(b)alt2	$2\left(\frac{1-\cos 2\theta}{1+\cos 2\theta}\right)-2=7\sec\theta \Rightarrow 2\tan^2\theta-2=7\sec\theta$	M1
	$2\tan^2\theta-2=7\sqrt{1+\tan^2\theta}$ $\left(2\tan^2\theta-2\right)^2=\left(7\sqrt{1+\tan^2\theta}\right)^2\Rightarrow 4\tan^4\theta-8\tan^2\theta+4=49\left(1+\tan^2\theta\right)$	M1
	$4\tan^4\theta-57\tan^2\theta-45=0$	A1
	$\left(4\tan^2\theta+3\right)\left(\tan^2\theta-15\right)=0\Rightarrow\tan^2\theta=15$	
	$\tan\theta=\sqrt{15}\Rightarrow\theta=...$	M1
	$\Rightarrow \theta=75.5°,-75.5°$	A1A1
		(6)

(a)
M1: Uses a correct double angle identity on the numerator or denominator **and applies this to the fraction**.
dM1: Uses correct double angle identities in the numerator and denominator leading to an expression of the form $\frac{a\sin^2 x}{a\cos^2 x}$
A1*: Completely correct solution. The variables must be consistent and do not accept expressions of the form '$\frac{\sin^2}{\cos^2}=\tan^2$' within the proof. If their working necessitates the appearance of the 2's in the numerator and denominator and they are not shown, this mark can be withheld – see examples.

(a) Alt1:
M1: Uses the identity $\tan^2 x=\frac{\sin^2 x}{\cos^2 x}$
dM1: Uses any two correct double angle identities.

A1*: Completely correct solution. The variables must be consistent and do not accept expressions of the form '$\frac{\sin^2}{\cos^2}=\tan^2$' within the proof.

(a) Alt 2:
M1: Multiplies both sides by the denominator of the lhs and uses any two correct double angle identities
dM1: Uses any two correct double angle identities.
A1: Obtains a correct identity and makes a conclusion.
See main scheme for some other varieties and the marks to award

(b) **Inc. Alt 1**

M1: Obtains an equation of the form $A\tan^2\theta - 2 = 7\sec\theta$ or $A\tan^2\theta - 2 = \frac{7}{\cos\theta}$

M1: Attempts to use the trig identity $\tan^2\theta = \pm\sec^2\theta \pm 1$ to produce a quadratic equation in $\sec\theta$ or attempts to use $\tan^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$ and $\sin^2\theta = \pm 1 \pm \cos^2\theta$ to produce a quadratic in $\cos\theta$.

A1: Correct 3TQ = 0. Either $2\sec^2\theta - 7\sec\theta - 4 = 0$ or $4\cos^2\theta + 7\cos\theta - 2 = 0$ or equivalent

M1: Correct method of solving 3TQ = 0 in either $\sec\theta$ or $\cos\theta$ **AND** using arccos in producing at least one answer for θ. You may need to check the roots of their quadratic if no working is seen and if the roots are incorrect and no working is shown, score M0.

A1: One of awrt $\theta = 75.5°, -75.5°$

A1: Both of awrt $\theta = 75.5°, -75.5°$

In an otherwise correct solution, deduct the final mark for extra answers in range. Ignore answers outside the range.

(b) **Alt 2**

M1: Obtains an equation of the form $A\tan^2\theta - 2 = 7\sec\theta$ or $A\tan^2\theta - 2 = \frac{7}{\cos\theta}$

M1: Attempts to use the trig identity $\tan^2\theta = \pm\sec^2\theta \pm 1$ and squares to produce a quadratic equation in $\tan^2\theta$

A1: Correct 3TQ = 0. $4\tan^4\theta - 57\tan^2\theta - 45 = 0$ or equivalent

M1: Correct method of solving 3TQ = 0 **AND** using arctan after square root in producing at least one answer for θ. You may need to check the roots of their quadratic if no working is seen and if the roots are incorrect and no working is shown, score M0.

A1: One of awrt $\theta = 75.5°, -75.5°$

A1: Both of awrt $\theta = 75.5°, -75.5°$

For answers in radians (awrt 1.3, -1.3) deduct the final A mark.

In an otherwise correct solution, deduct the final mark for extra answers in range. Ignore answers outside the range.

Part (b) Note: If the quadratic (in sec or cos) is incorrect but fortuitously leads to the correct answers e.g. from factors of $(\sec\theta - 4)$ or $(4\cos\theta - 1)$ then the final A mark can be withheld. If the quadratic (in sec or cos) is correct but in their factorisation the $(\sec\theta - 4)$ or $(4\cos\theta - 1)$ is correct and the other factor incorrect then the final A mark can be withheld if they proceed to obtain the correct angles.
M1: Obtains an equation of the form $A\tan^2\theta - 2 = 7\sec\theta$ or $A\tan^2\theta - 2 = \frac{7}{\cos\theta}$ (or a method using identities (allow sign errors) to obtain an equation in terms of single angles) M1: Uses identities (allow sign errors) to produce an equation in terms of a single trig. function. A1: Correct equation M1: Solves to obtain at least one value A1: One of awrt $\theta = 75.5°, -75.5°$ A1: Both of awrt $\theta = 75.5°, -75.5°$

Q11.

Question Number	Scheme	Notes	Marks
	$V = \frac{1}{3}\pi h^2(90 - h) = 30\pi h^2 - \frac{1}{3}\pi h^3$; $\frac{dV}{dt} = 180$		
	$\frac{dV}{dh} = 60\pi h - \pi h^2$	$\left\{\frac{dV}{dh} =\right\} \pm\alpha h \pm \beta h^2,\ \alpha \neq 0,\ \beta \neq 0$	M1
		$60\pi h - \pi h^2$ Can be simplified or un-simplified.	A1
	$\left\{\frac{dV}{dh} \times \frac{dh}{dt} = \frac{dV}{dt} \Rightarrow\right\}\ \left(60\pi h - \pi h^2\right)\frac{dh}{dt} = 180$ $\left\{\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} \Rightarrow\right\}\ \frac{dh}{dt} = 180 \times \frac{1}{60\pi h - \pi h^2}$	$\left(\text{their } \frac{dV}{dh}\right) \times \frac{dh}{dt} = 180$ or $180 \div$ their $\frac{dV}{dh}$ This is for a correct **application** of the chain rule and not for just quoting a correct chain rule.	M1
	When $h = 15$, $\left\{\frac{dh}{dt} =\right\} \frac{1}{60\pi(15) - \pi(15)^2} \times 180\ \left\{= \frac{4}{15\pi}\right\}$	**Dependent on the previous M mark.** Substitutes $h = 15$ into an expression which is a result of a quotient (or their rearranged quotient) of their $\frac{dV}{dh}$ and 180. May be implied by awrt 0.08 or 0.09.	dM1
	$\left\{\frac{dh}{dt} = 0.0848826... \Rightarrow\right\} \frac{dh}{dt} = \underline{0.085}\ (\text{cm s}^{-1})$ (2 sf)	**Awrt** 0.085 or allow $\frac{4}{15\pi}$ oe (and isw if necessary)	A1 cao
			[5]
			5
	<u>Alternative Method for the first M1A1</u>		
	Product rule: $\left\{\begin{matrix} u = \frac{1}{3}\pi h^2 & v = 90 - h \\ \frac{du}{dh} = \frac{2}{3}\pi h & \frac{dv}{dh} = -1 \end{matrix}\right\}$		
	$\frac{dV}{dh} = \frac{2}{3}\pi h(90 - h) + \frac{1}{3}\pi h^2(-1)$	$\left\{\frac{dV}{dh} =\right\} \pm\alpha h(90 - h) \pm \beta h^2(-1),\ \alpha \neq 0,\ \beta \neq 0$ Can be simplified or un-simplified.	M1
		$\frac{2}{3}\pi h(90 - h) + \frac{1}{3}\pi h^2(-1)$ Can be simplified or un-simplified.	A1

Question Notes

	Note	$\frac{dV}{dh}$ does not have to be explicitly stated for the 1st M1 and/or the 1st A1 but it should be clear that they are differentiating their V.
	Note	$V = \frac{1}{3}\pi h^2(90 - h) \Rightarrow \frac{dV}{dh} = \frac{2}{3}\pi h(90 - h)$ scores M0A0 even though it satisfies the conditions for the derivative.

Q12.

Question	Scheme	Marks
(a)	$\frac{1}{2}\times 3^2 \times \alpha = 7.2 \Rightarrow \alpha = ...$ or $\frac{1}{2}\times 3^2 \times 1.6 = 7.2 \Rightarrow \alpha = 1.6$	M1
	$\alpha = 1.6*$	A1*
		(2)
(b)(i)	Angle $COA = \frac{1}{2}(2\pi - 1.6)(= 2.34...)$ $(\approx 134°)$	M1
	Area $COA = \frac{1}{2}\times 5 \times 3\sin("2.34")$ $(= 5.38...)$	M1
	Total Area $= 2\times\frac{1}{2}\times 5\times 3\sin("2.34") + 7.2$	dM1
	= 18 (cm^2) Awrt 18 (cm^2) (Ans = 17.96)	A1
(ii)	Arc $AB = 3\times 1.6 (= 4.8)$	B1
	$(AC^2 =)\ 5^2 + 3^2 - 2\times 5\times 3\cos("2.34")$	M1
	Total perimeter $= 2\times\sqrt{5^2 + 3^2 - 2\times 5\times 3\cos("2.34")} + 3\times 1.6$	dM1
	= Awrt 19.6 (cm)	A1
		(8)
Alt (b)(i)	$AB = 2\times 3\sin 0.8$	M1
	$ON = 3\cos 0.8$	M1
	Total Area $= \frac{1}{2}(5 + ON)\times AB + 7.2 - \frac{1}{2}\times 3\cos 0.8\times 2\times 3\sin 0.8$	dM1
	= 18 (cm^2) Awrt 18 (cm^2) (Ans = 17.96)	A1
		(10 marks)

Notes

(a)

M1 Uses a correct sector area formula and 7.2 to find the value for α. They should show the values embedded in the equation and proceed to find a value for α.
Alternatively, substitutes in $\alpha = 1.6$ into the area of a sector formula and achieves 7.2.

A1* Correct proof starting with $\frac{1}{2}\times 3^2\times\alpha = 7.2$ and at least one intermediate line of working and no errors. Eg $\frac{1}{2}\times 3^2\times\alpha = 7.2 \Rightarrow \alpha = \frac{7.2}{4.5} = 1.6$ scores M1A1
Alternatively, they must conclude that $\alpha = 1.6$ or if there is a preamble then there should be some form of completion which could be a tick, QED etc.
If they use a different variable such as θ they must state/link somewhere that $\alpha = 1.6$

(b)(i) **Mark both (i) and (ii) together. If no angle calculation is seen then use what they think is their angle *COA* in bi and bii. Beware of values on the diagram that may imply a method.**

M1 $\frac{1}{2}(2\pi - 1.6)$ Correct method for angle *COA*. Sight of awrt 2.34 is sufficient to score this mark and may be on the diagram. (May also be implied by 134°)

M1 Uses a correct method for the area of triangle *COA* or *COB*. It is sufficient to see the values embedded in the expression such as $\frac{1}{2}\times 5\times 3\sin("2.34")$ $(= \text{awrt } 5.38 / 5.39)$. Angle may be in degrees. If they state $\frac{1}{2}ab\sin C$ oe but embed values as $\frac{1}{2}abC$ condone as a slip for M1.

dM1 Fully correct strategy for the area. It is dependent on the previous method mark so allow if their angle *COA* is incorrect. Look for $2\times$area of triangle $COA + 7.2$. Embedded values are sufficient.

A1 awrt 18 (cm^2) **Must come from a correct method**

Alt b(i)

M1 Find the length $AB = 2\times 3\sin 0.8$ (awrt 4.30)

M1 Finds the length *ON* where *N* is the midpoint of *AB* (awrt 2.09)

dM1 Fully correct strategy for the area. Look for $7.2 +$ area of triangle $ABC -$ area of triangle AOB

A1 awrt 18 (cm^2) **Must come from a correct method**

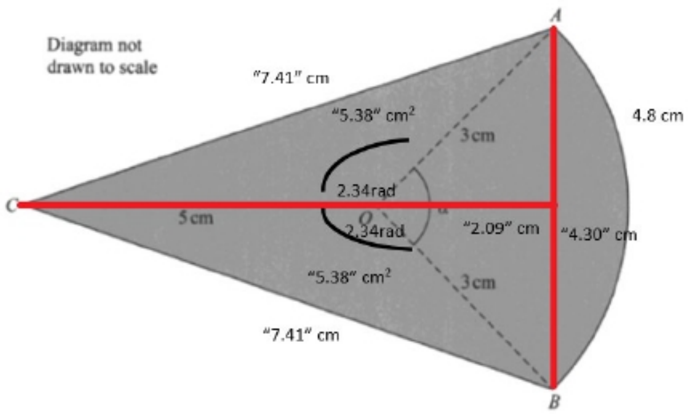
(ii)

B1 Correct expression or value for the arc length (4.8)

M1 Uses a correct method for AC^2, AC, CB^2 or CB. Embedded values in the associated formula is sufficient or sight of awrt 54.9 or awrt 7.41 would imply this mark. (Angle in degrees $\approx 134°$) For this mark condone candidates confusing AC^2/CB^2 and AC/CB.

dM1 Total perimeter $= 2\times\sqrt{5^2+3^2-2\times 5\times 3\cos("2.34")} + 3\times 1.6$. It is dependent on the B and the M marks and they must have remembered to square root AC^2 or CB^2.

A1 awrt 19.6 (cm) (Ans = 19.619) **Must come from a correct method**



Q13.

Question Number	Scheme	Marks
	Assume that there exists a number m such that when m^3 is even, m is odd	B1
	If m is odd then $m = 2p+1$ (where p is an integer) and $m^3 = (2p+1)^3 = ...$	M1
	$= 8p^3 + 12p^2 + 6p + 1$	A1
	$2\times(4p^3 + 6p^2 + 3p) + 1$ is odd and hence we have a contradiction so if n^3 is even, then n is even.	A1
		(4)
		(4 marks)

B1: For setting up the contradiction.

Eg Assume that there exists a number m such that when m^3 is even, m is odd

Condone a contra-positive statement here

"Assume that there exists a number m such that when m^3 is even, m is not even"

As a minimum accept "assume if m^3 is even then m is odd."

Condone the other way around " assume if n is odd then n^3 is even"

M1: Attempts to cube an odd number. Accept an attempt at $(2p+1)^3$, $(2p-1)^3$

Look for $(2p+1)^3 = ...p^3.....$

A1: $(2p+1)^3 = 8p^3 + 12p^2 + 6p + 1$ or simplified equivalent such as $2\times(4p^3 + 6p^2 + 3p) + 1$.

For $(2p-1)^3 = 8p^3 - 12p^2 + 6p - 1$ or equivalent such as $2\times(4p^3 - 6p^2 + 3p - 1) + 1$

A1: For a fully correct proof. Requires correct calculations with reason and conclusion

E.g. 1 Correct calculations $(2p+1)^3 = 8p^3 + 12p^2 + 6p + 1 =$

Reason (even +1) = odd

Conclusion "hence we have a contradiction, so if n^3 is even, then n is even."

E.g. 2 Correct calculations $(2p+1)^3 = 8p^3 + 12p^2 + 6p + 1$

Reason $= 2\times(4p^3 + 6p^2 + 3p) + 1$ = odd

Conclusion "this is contradiction, so proven."

E.g. 3 Correct calculations $(2p-1)^3 = 8p^3 - 12p^2 + 6p - 1$

Reason = $8p^3 - 12p^2 + 6p$ is even so $8p^3 - 12p^2 + 6p - 1$ is odd

Conclusion: So if n^3 is even then n must be even

Note that B0 M1 A1 A1 is possible

Q14.

The correct answer, unless clearly obtained by an incorrect method, should be taken to imply a correct method.

Question	Working	Answer	Mark	Notes
(a) (i)		$\mathbf{b} - 2\mathbf{a}$	3	B1
(ii)		$\frac{2}{3}\mathbf{b} - \frac{4}{3}\mathbf{a}$		B1 oe eg. $\frac{2}{3}(-2\mathbf{a} + \mathbf{b})$ Allow ft from (i)
(iii)		$\frac{2}{3}\mathbf{b} - \frac{1}{3}\mathbf{a}$		B1 oe. eg. $\mathbf{a} + \frac{2}{3}(-2\mathbf{a} + \mathbf{b})$ Allow ft from (ii)
(b)			2	M1 for $\overrightarrow{WY} = -\mathbf{a} + 2\mathbf{b}$ oe **or** $\overrightarrow{XY} = \frac{2}{3}(-\mathbf{a} + 2\mathbf{b})$ oe Allow ft from (a)
		shown		A1 for conclusion using correct vectors eg. $\overrightarrow{WY} = 2\mathbf{b} - \mathbf{a}$ $\overrightarrow{XY} = \frac{2}{3}(-\mathbf{a} + 2\mathbf{b})$ $\overrightarrow{XY} = \frac{2}{3}\overrightarrow{WY}$
				Total 5 marks

Q15.

6a) $f(x) = x^2 + 4x + 1$

$= (x + 2)^2 - 4 + 1$

$= (x + 2)^2 - 3$

Min point : $(-2, -3)$ $\quad f(x) \geq -3$

b) It is a many to one function. (many to one functions do not have an inverse)